Pitch Talk
Abstract Cores in Implicit Hitting Set MaxSat Solving

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VELUT ARBOR ÆVO

Motivation

- MaxSAT - optimization extension of Boolean Satisfiability
- Competitive approach for solving many real-world optimisation problems.
  - last two evaluations had benchmark submissions from 17 new domains.
- IHS solvers central in real-world MaxSAT solving
  - Decouple MaxSAT into core-extraction (SAT-solving) and optimization (IP-solving),
  - avoid increasing complexity of SAT calls but,
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Potential solution: Abstract cores

Abstract Core: a compact representation of a (potentially) exponential number of normal cores

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$$\mathcal{F}_H = \left\{ \text{CNF}\left(\sum_{i=1}^{n} b_i \geq r \right) \right\}$$

$$\mathcal{F}_S = \{(\neg b_1), \ldots, (\neg b_n)\}$$

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Abstract Core: a compact representation of a (potentially) exponential number of normal cores

$$\mathcal{F}_H = \left\{ \text{CNF}\left(\sum_{i=1}^{\mathbf{8}} b_i \geq \mathbf{4} \right) \right\}$$

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$$(b_{i_1} \vee b_{i_2} \vee b_{i_3} \vee b_{i_4} \vee b_{i_5})$$

is a core for any i_1, i_2, i_3, i_4, i_5.

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$(b_3 \lor b_5 \lor b_6 \lor b_7 \lor b_8)$
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Idea:

Consider: $AB = \{b_1, \ldots, b_5\}$

Introduce abstraction variable: $s^{AB}[i]$

Define: $s^{AB}[i] \leftrightarrow \left(\sum_{b \in AB} b \geq i\right)$

Other definitions possible.
Summations successful in core-guided solvers

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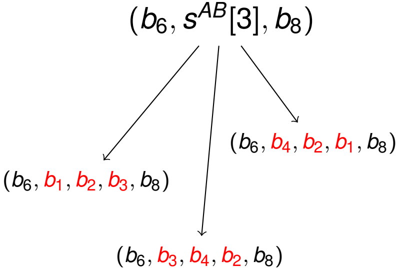

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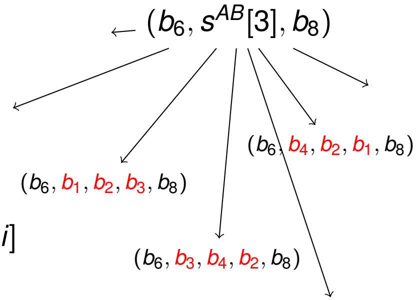

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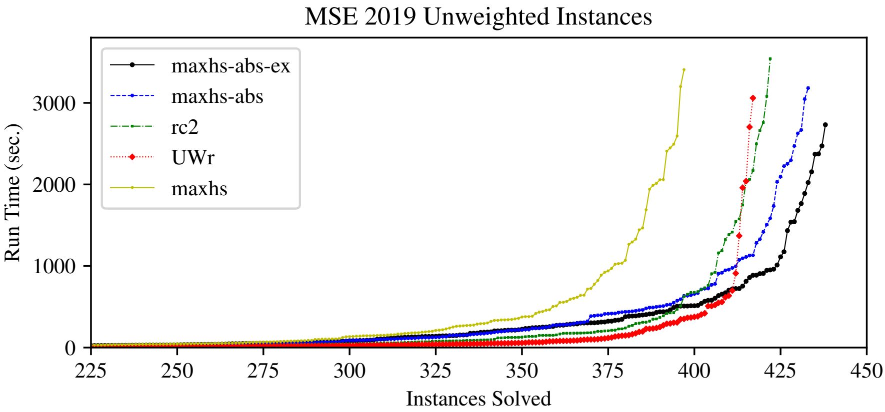

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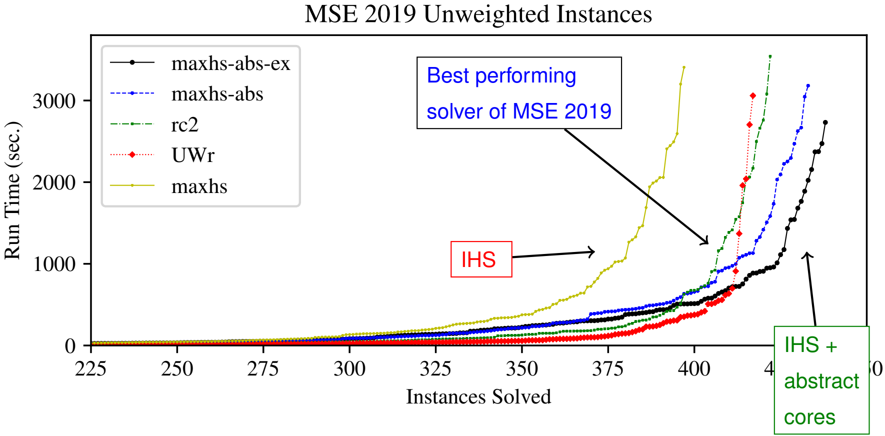

Any Questions?