Weight-Aware Core Extraction in SAT-Based MaxSAT Solving

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Motivation

- Constraint optimization, a generic, effective approach to several NP-hard problems in AI and industrial domains.
  - planning, debugging, and diagnosis, machine learning, systems biology etc.
- Maximum Satisfiability, a competitive paradigm alternative
- *Core-Guided MaxSAT*, most promising for large-scale industrial problems.
  - Extensive use of SAT-solvers.
  - Iteratively extract and rule out unsatisfiable sub-formulas
  - Increase complexity of working formula on each iteration

Argelich et al.

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Our Contributions

- Weight Aware Core Extraction (WCE)
  - Delay ruling out found cores, keep working formula simpler
  - Generic refinement to core-extraction and weight handling used in many solvers
  - Applicable to many modern MaxSAT algorithms
- In this paper, WCE for the PMRES algorithm Narodytska and Bacchus [2014].

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Outline

- Satisfiability (SAT) and Maximum Satisfiability (MaxSAT)
- Core Guided MaxSAT solving.
- Weight Aware Core Extraction (WCE)
- Implementing WCE
- Experimental Evaluation (with PMRES)
- Summary

Satisfiability

- Satisfiability (SAT): Given a set F of Boolean Constraints, decide if its satisfiable

$$F = \{(x \vee z), (x \vee y), (\neg x), (\neg z), (\neg y)\}$$

Weighted Partial Maximum Satisfiability

- (Weighted Partial) Maximum Satisfiability (MaxSAT):
  - Two sets of constraints: F_h, (hard clauses) and F_s (soft clauses, weighted)
  - Find assignment τ satisfying F_h and a maximum sum of weights of clauses in F_s.

$$F_h = \{(x \vee z), (x \vee y)\}$$

$$F_s = \{((\neg x), 3), ((\neg z), 1), ((\neg y), 1)\}$$

COST: 2

Weighted Partial Maximum Satisfiability

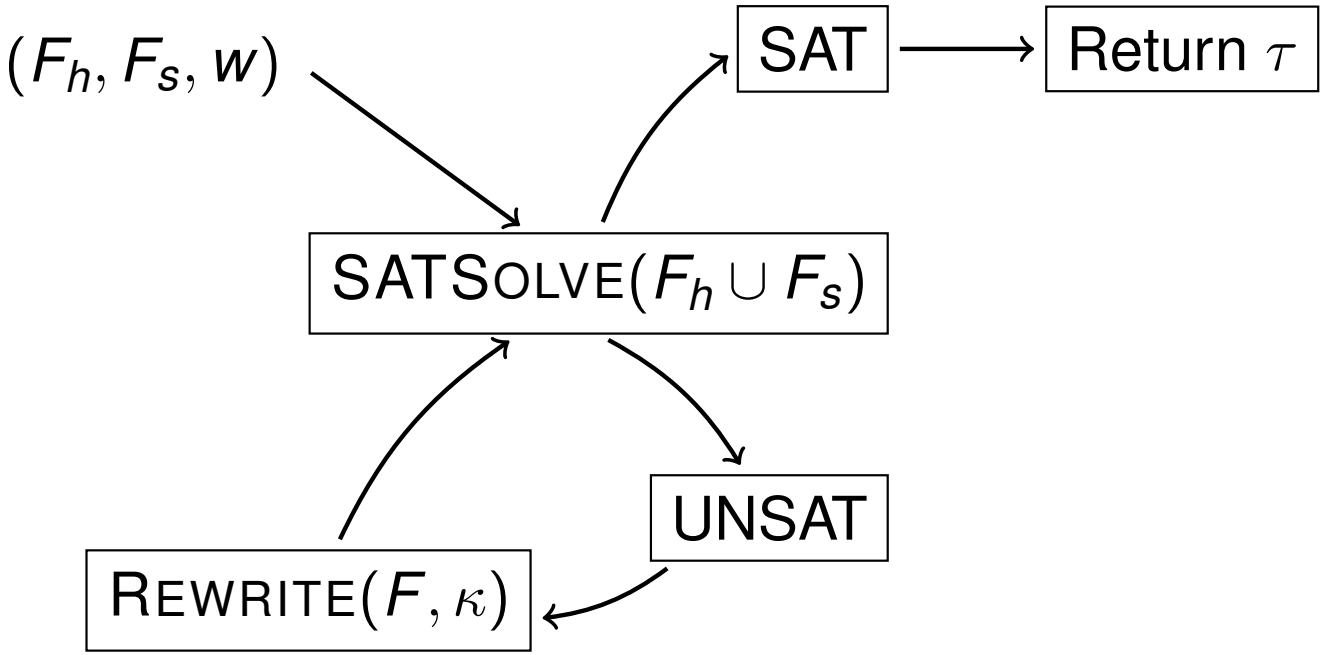
- A unsatisfiable core κ: a subset $\kappa \subseteq F_s$ s.t. $\kappa \wedge F_h$ is unsatisfiable.

$$F_h = \{(\mathbf{x} \vee \mathbf{z}), (x \vee y)$$

$$F_s = \{(\neg\mathbf{x}), (\neg\mathbf{z}), (\neg y)\}$$

Core Guided MaxSAT solving

Core Guided MaxSAT solving



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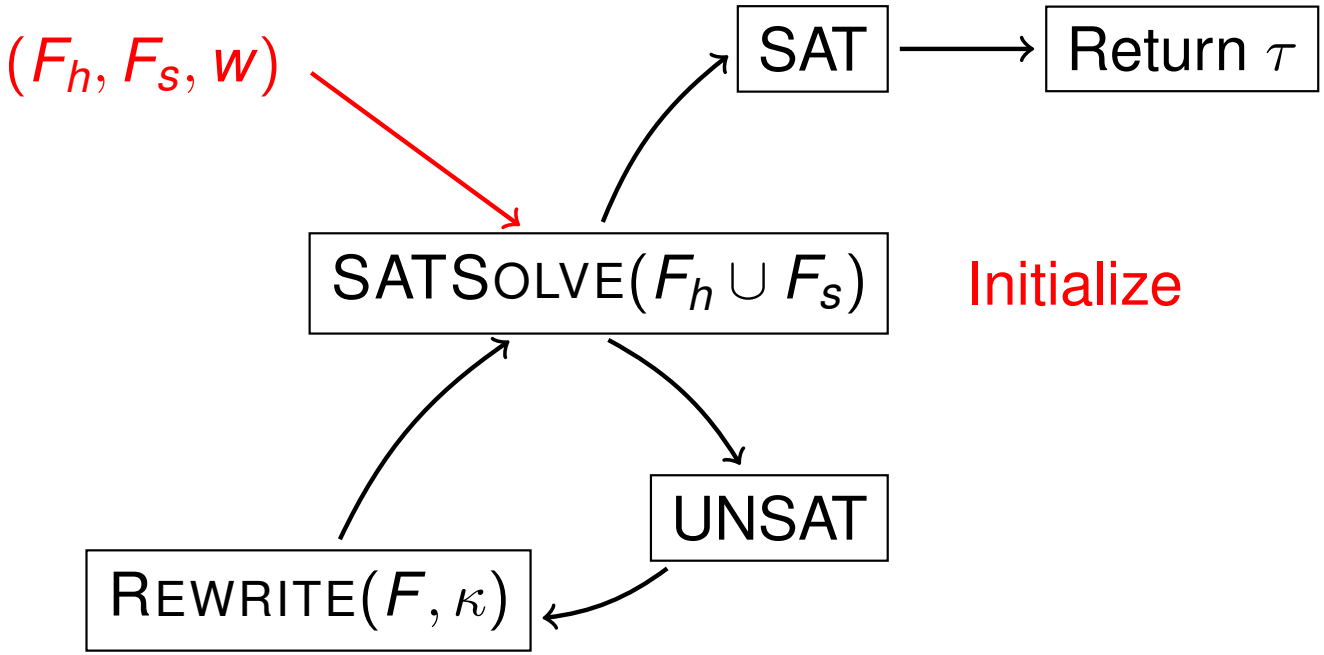

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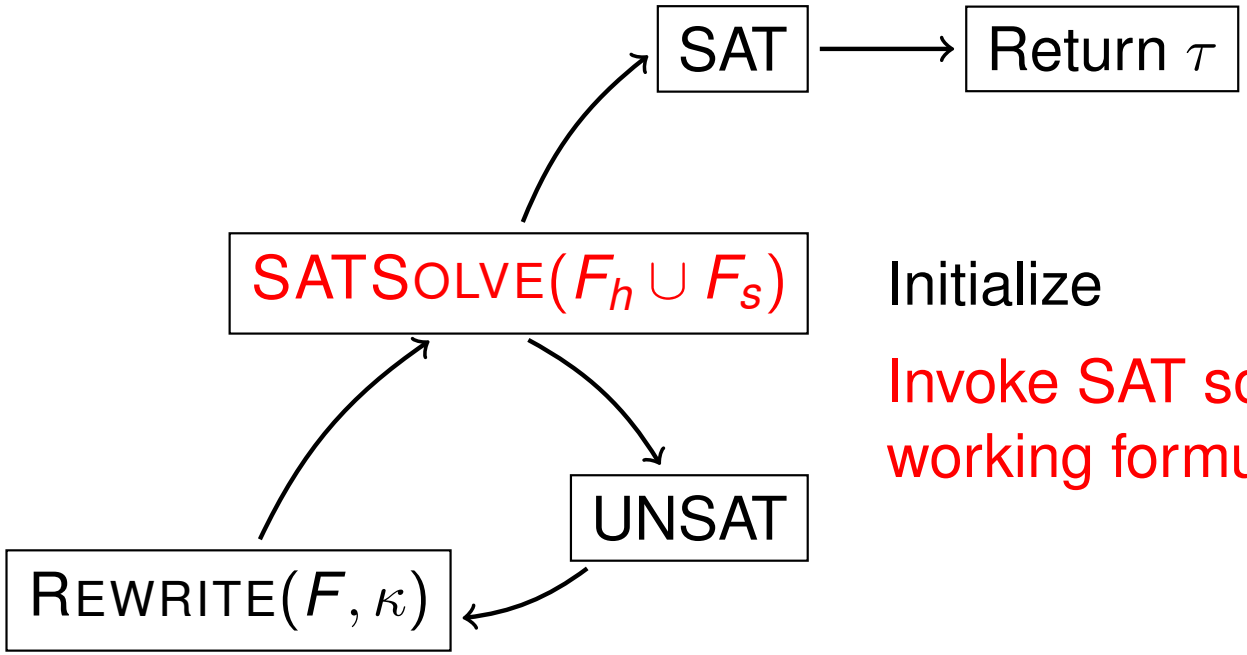
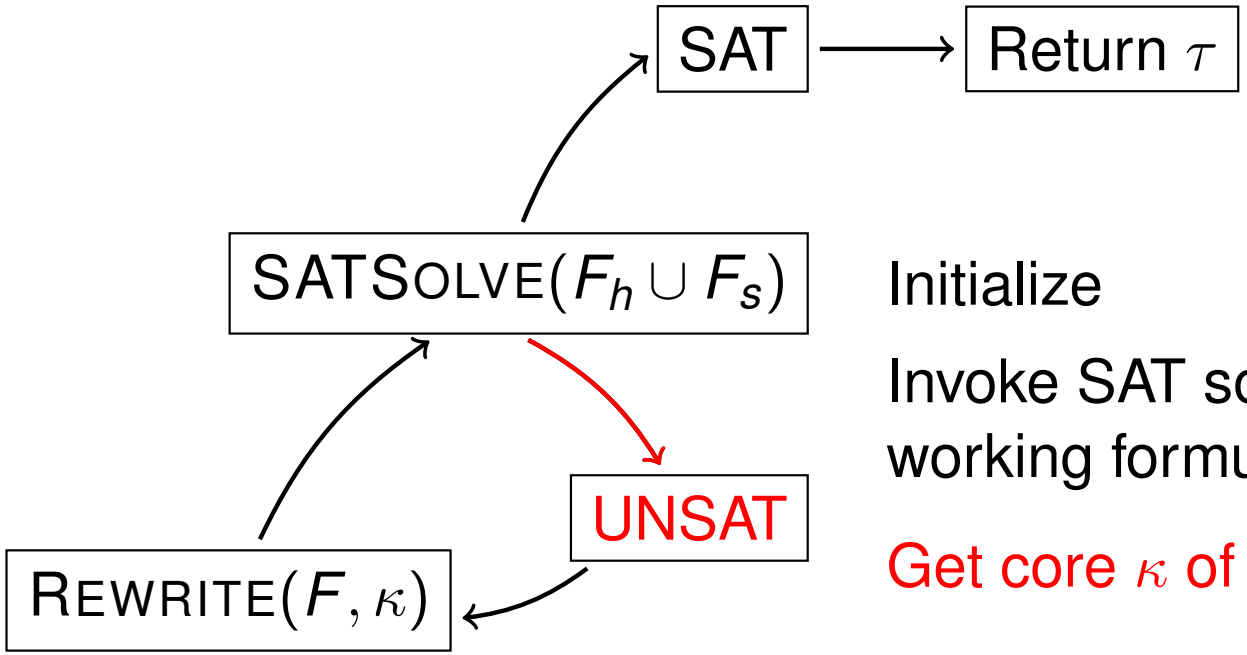


Initialize

Invoke SAT solver on working formula

Core Guided MaxSAT solving



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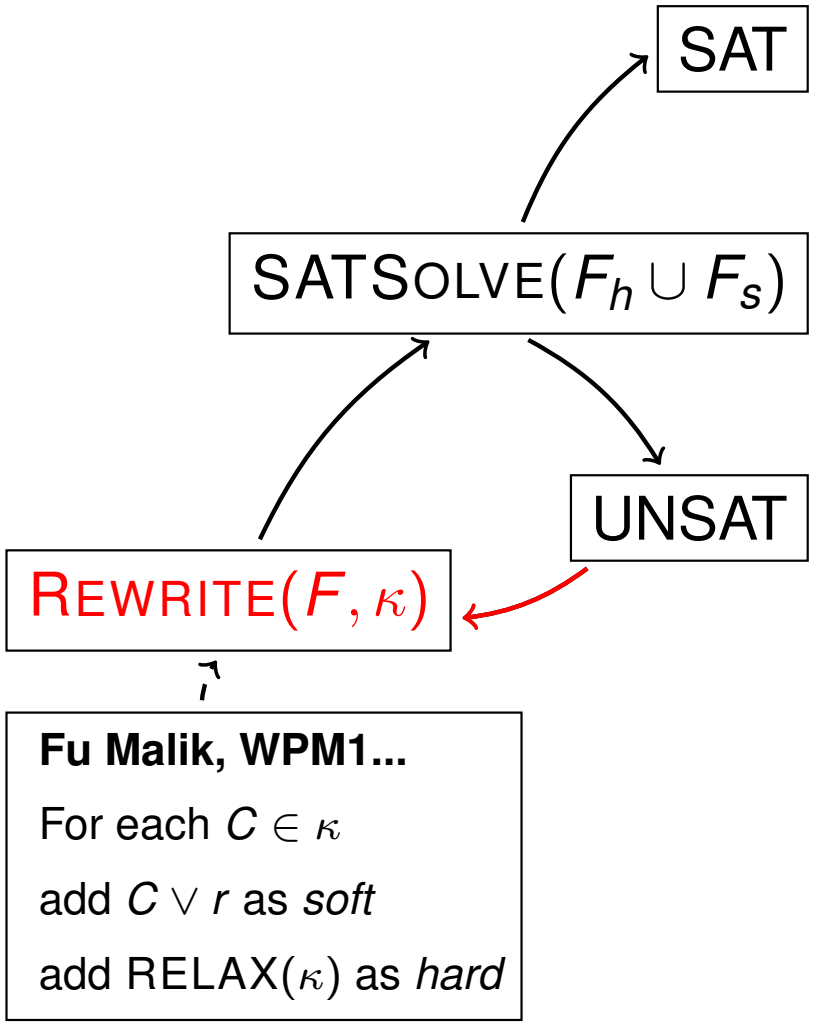
Get core κ of F

Core Guided MaxSAT solving Ansótegui et al. [2013]



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Get core κ of F

Compile Core into the formula

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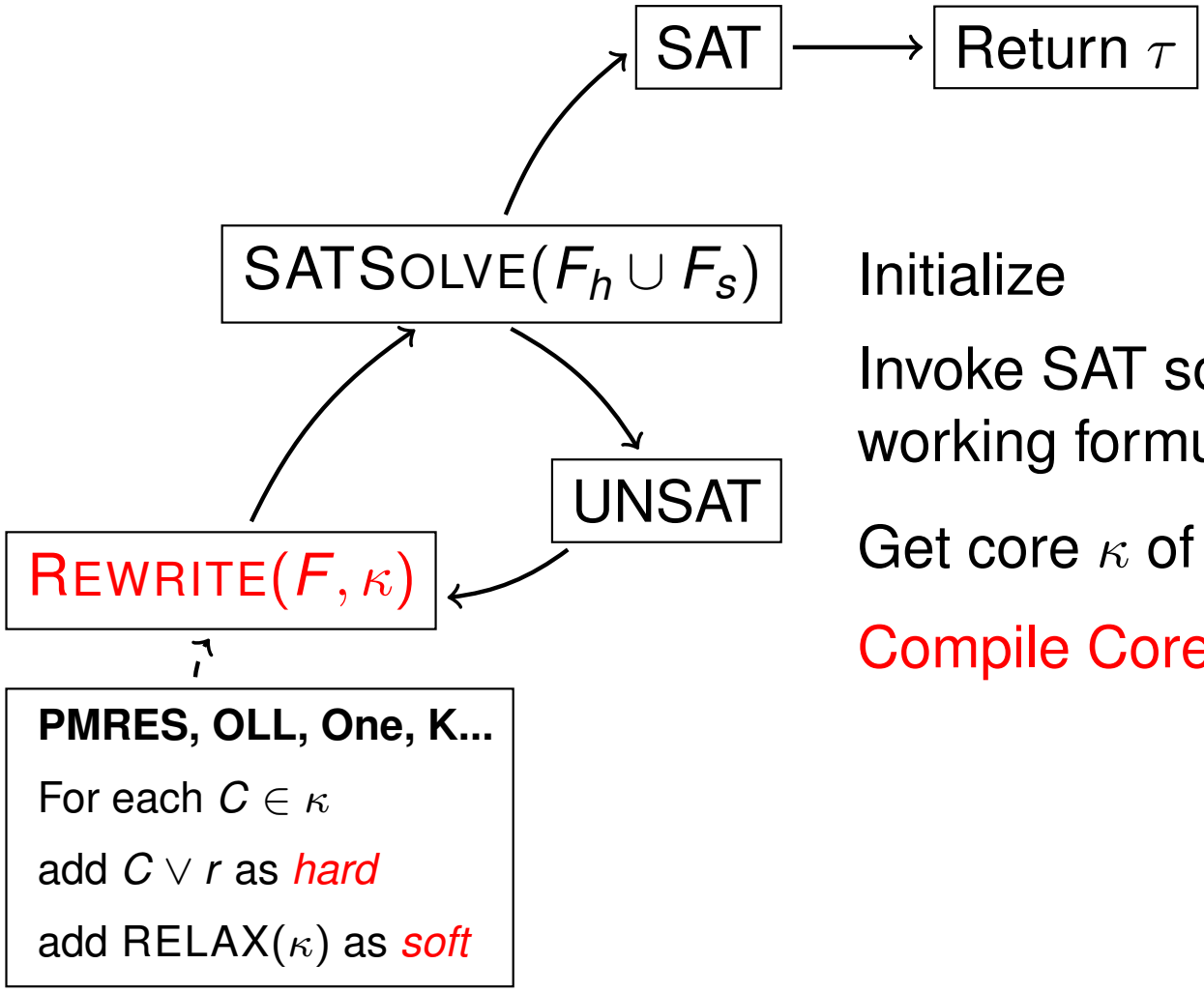

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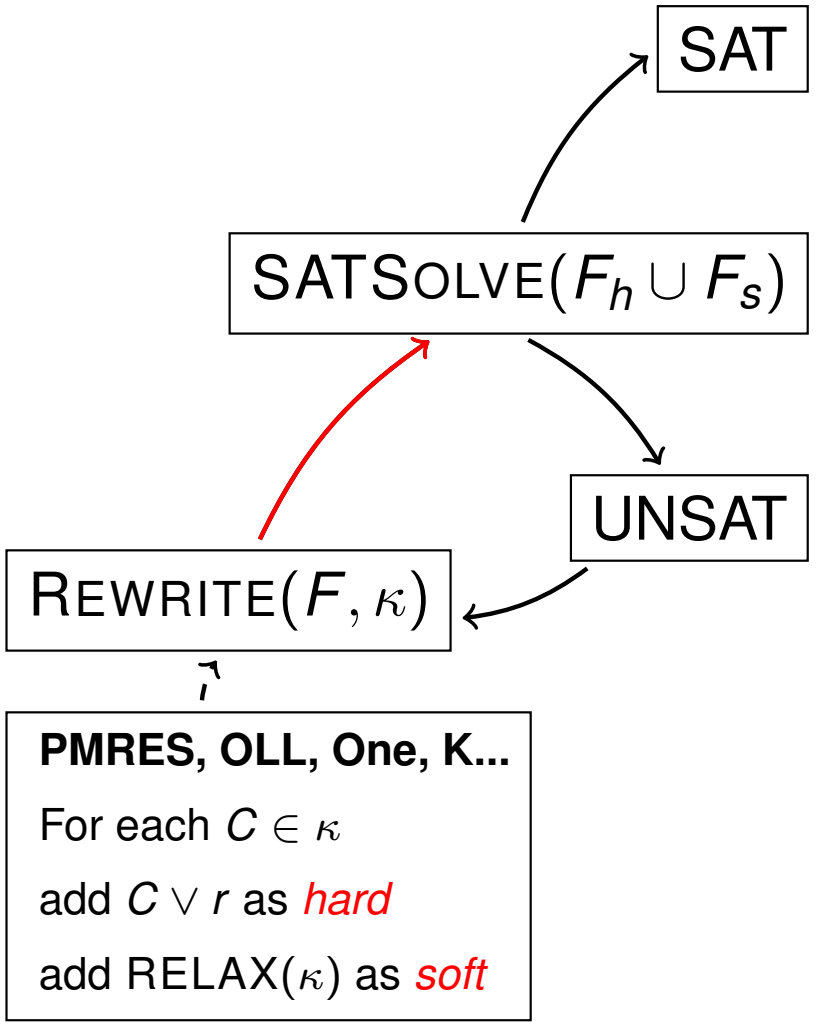


Initialize

Invoke SAT solver on working formula

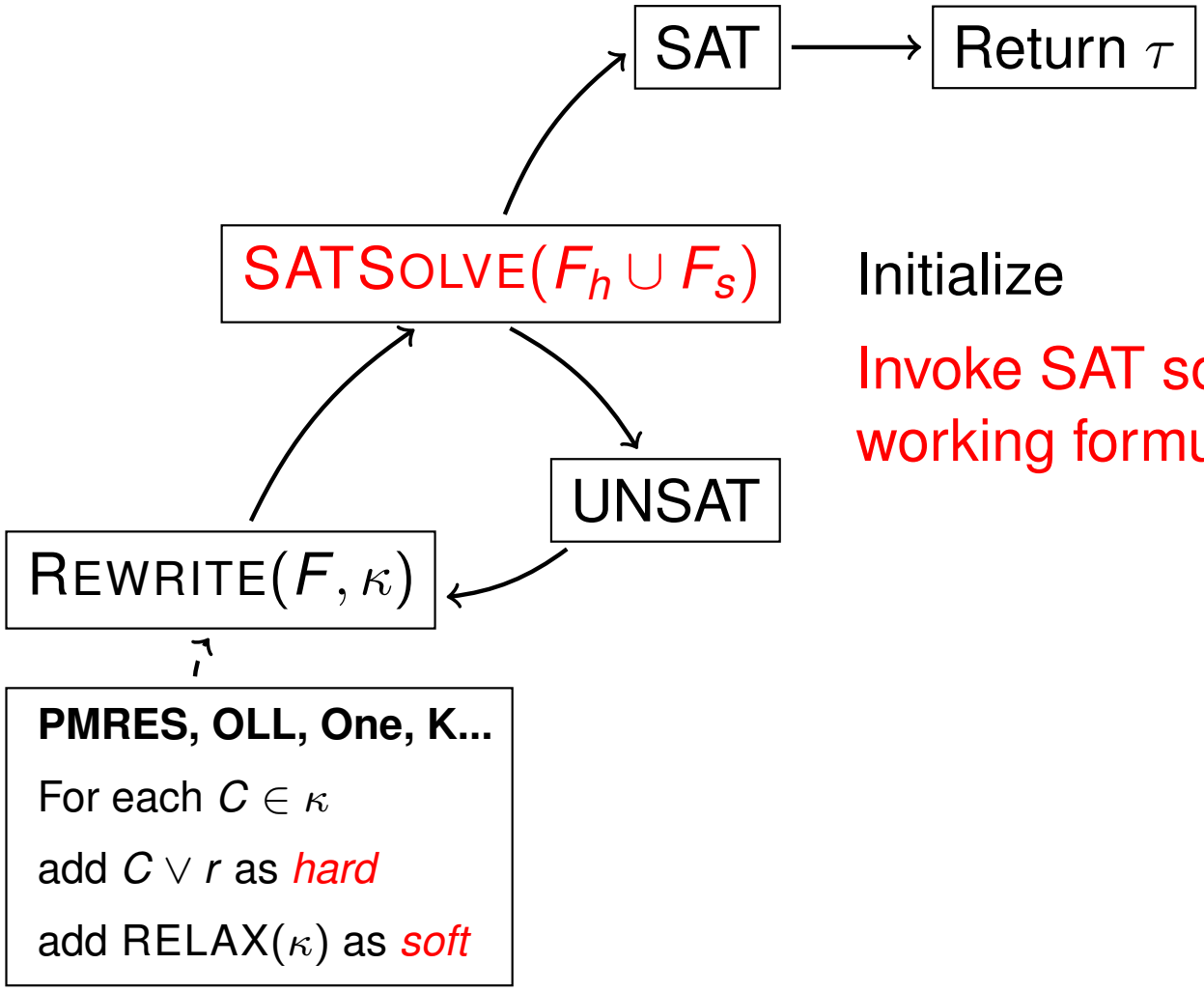
Get core κ of F

Compile Core into the formula

Reiterate

NOTE: Complexity increases

Core Guided MaxSAT solving



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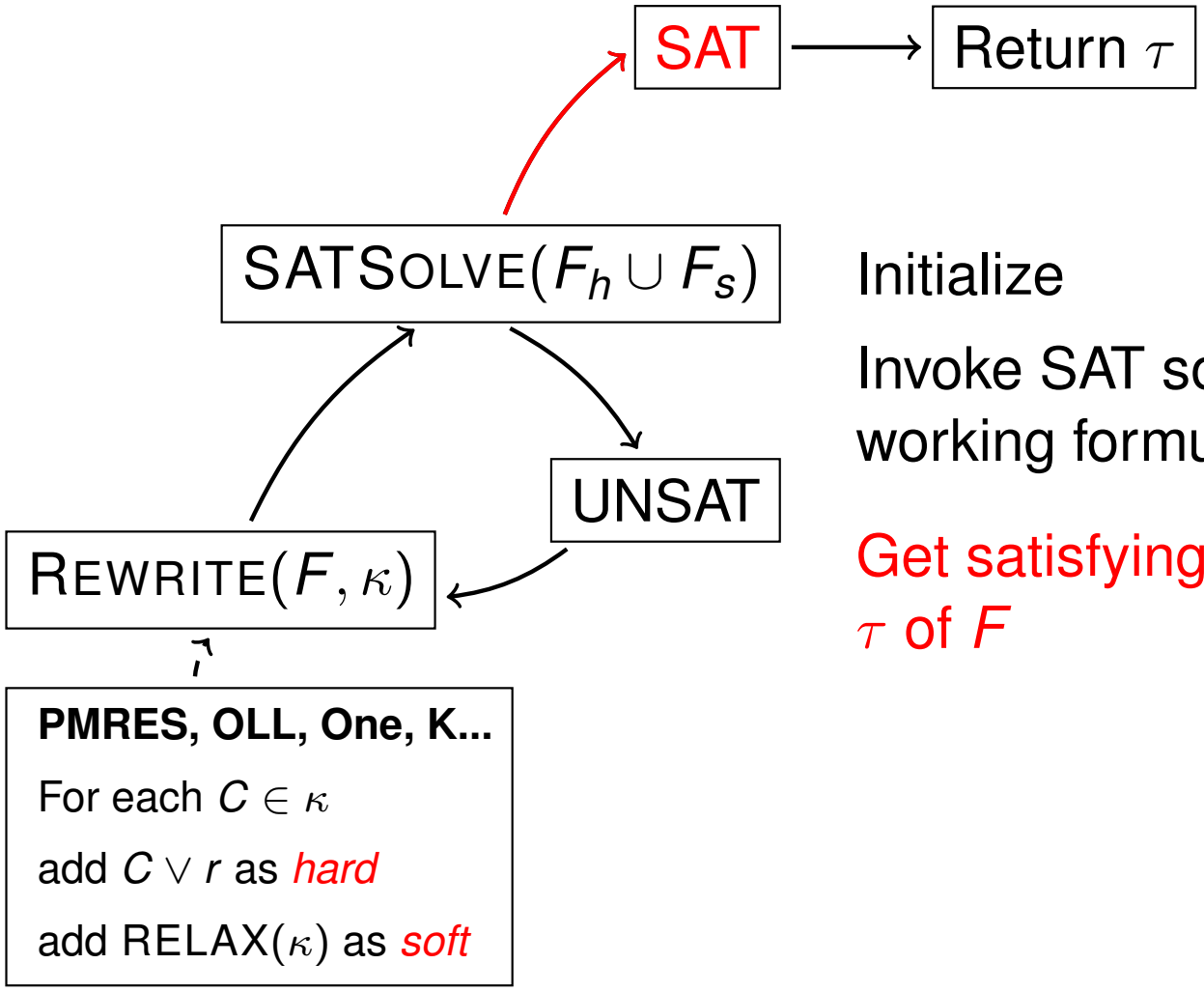
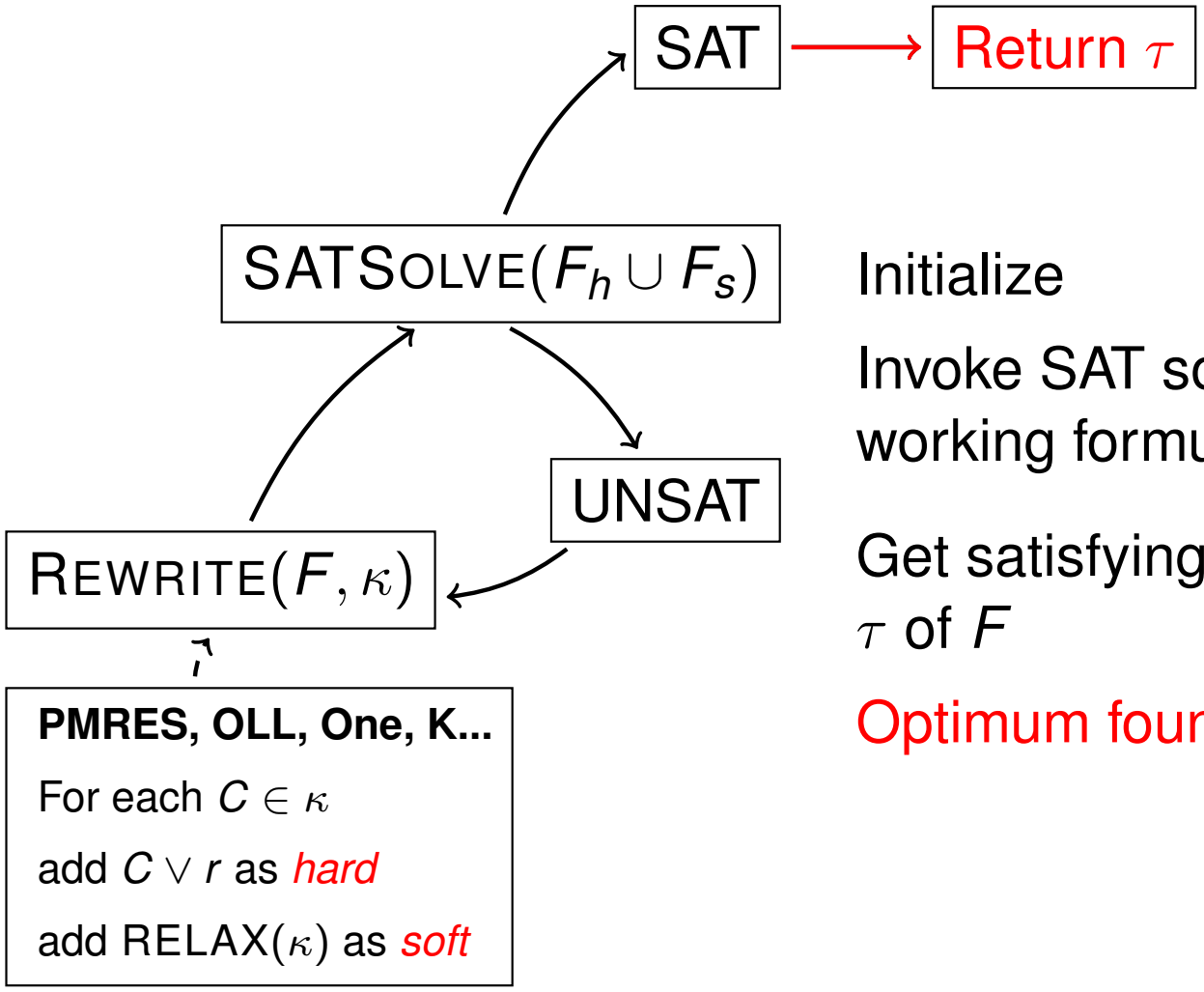


Initialize

Invoke SAT solver on working formula

Get satisfying assignment τ of F

Core Guided MaxSAT solving



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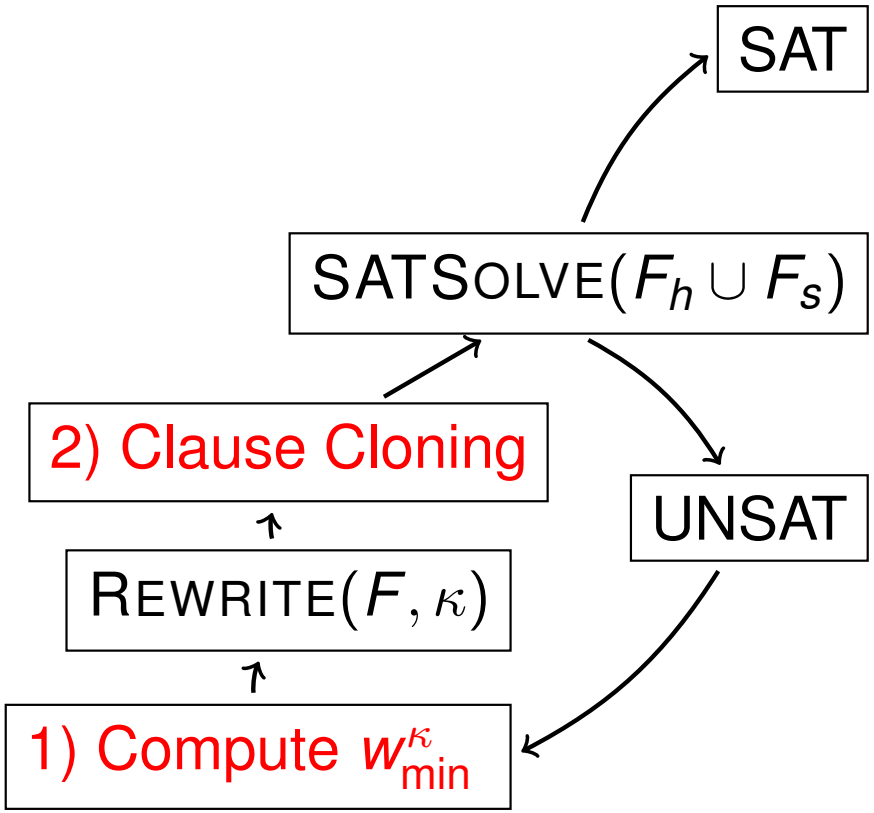
Get satisfying assignment τ of F

Optimum found

Core Guided **weighted** MaxSAT solving



Clause Cloning:

$w^{\kappa}_{\min} = \min\{w(C) \mid C \in \kappa\}$ (1)

Weight of new soft clauses $w^{\kappa}_{\min}$

For each $C \in \kappa$ s.t $w(C) > w^{\kappa}_{\min}$

Add clone of C with weight $w(C) - w^{\kappa}_{\min}$ (2)

NOTE: Sum of $w^{\kappa}_{\min}$

is a Lower Bound on solution cost

Example

$$F_h = \{(x \vee z), (x \vee y)\}$$
$$F_s = \{(\neg x, 3), (\neg y, 1), (\neg z, 1)\}$$

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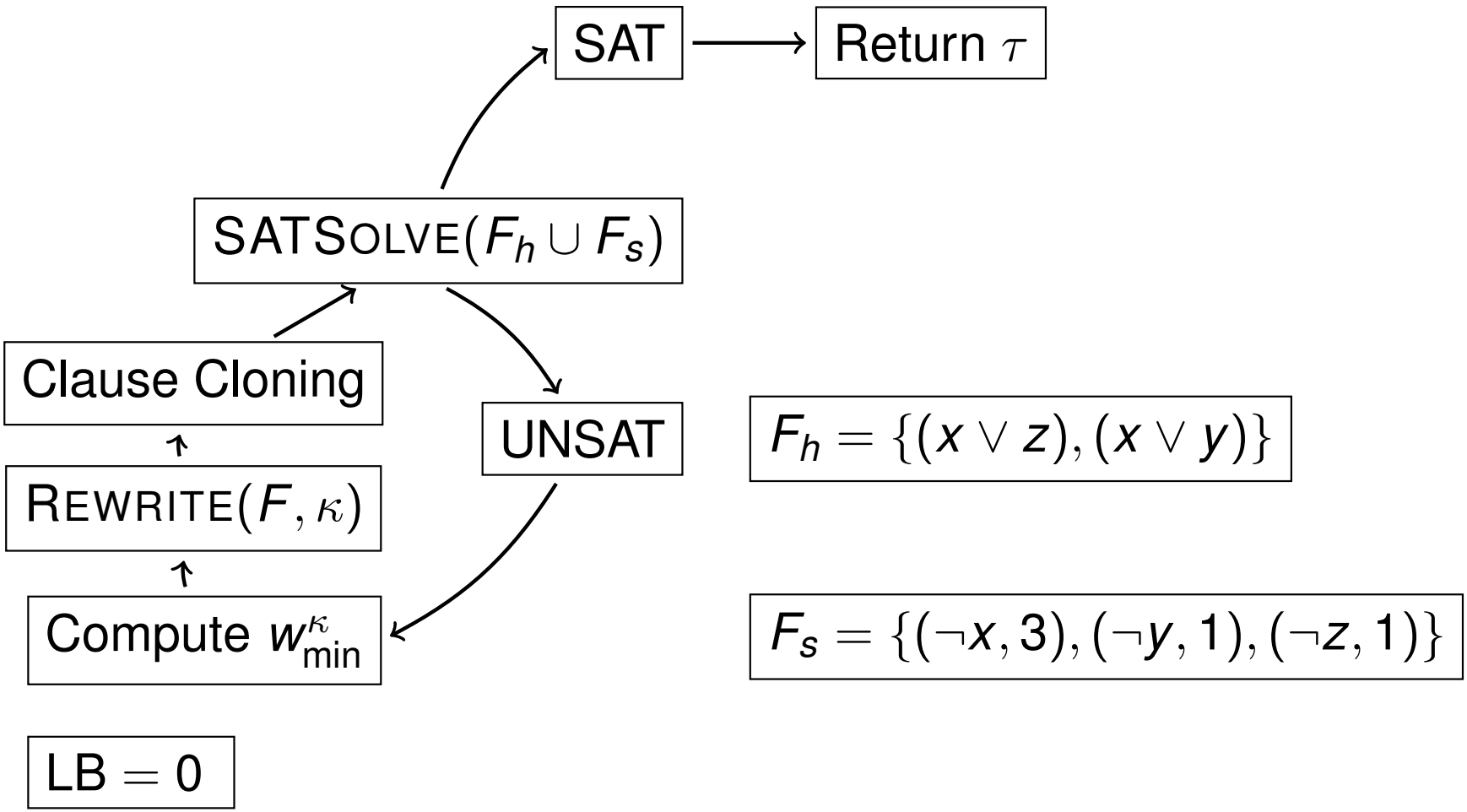
SAT
Return τ
SATSOLVE($F_h \cup F_s$)
Clause Cloning
UNSAT
REWRITE(F, κ)
Compute w^{κ}_{min}
LB $= 0$
$F_h = \{(x \vee z), (x \vee y)\}$
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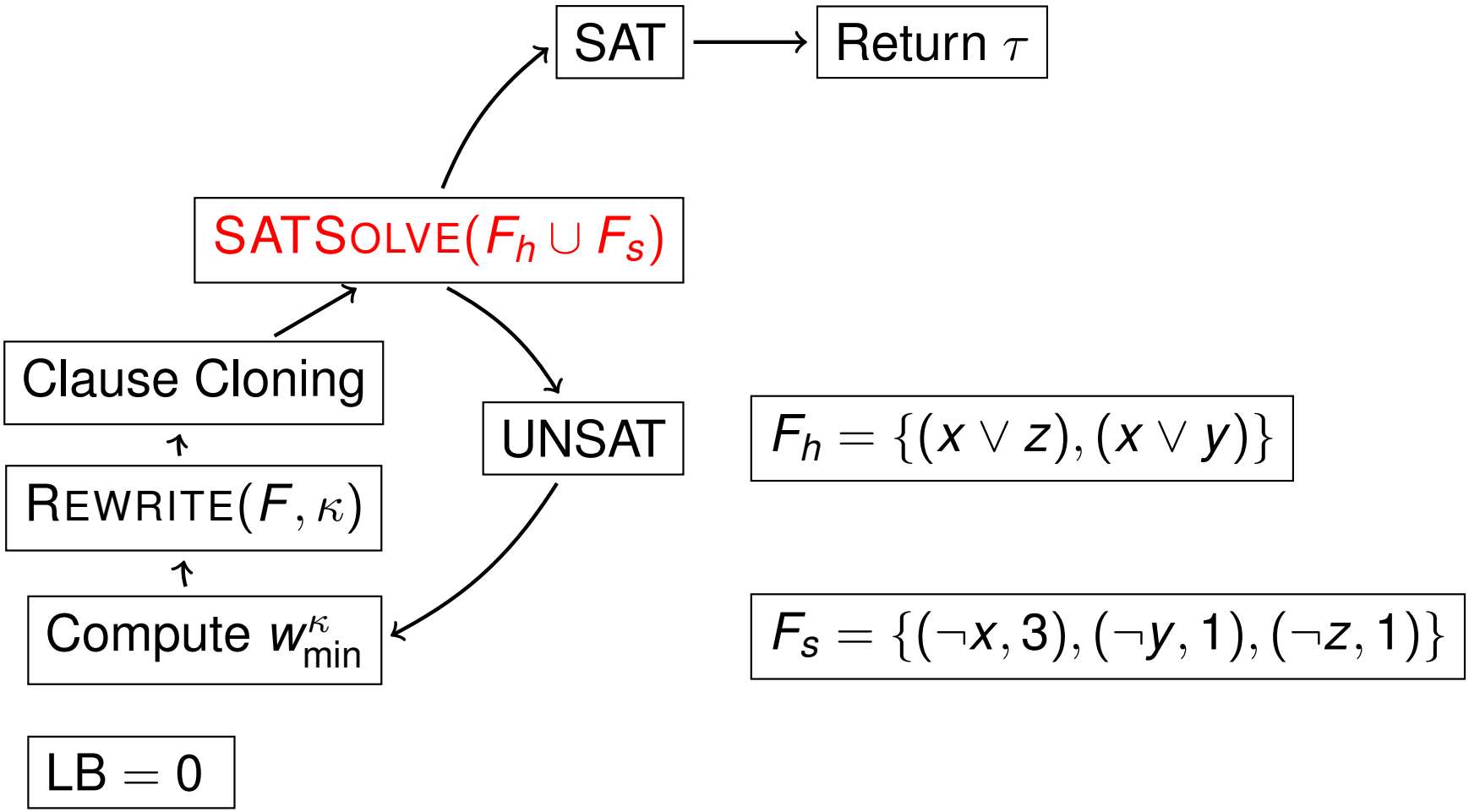
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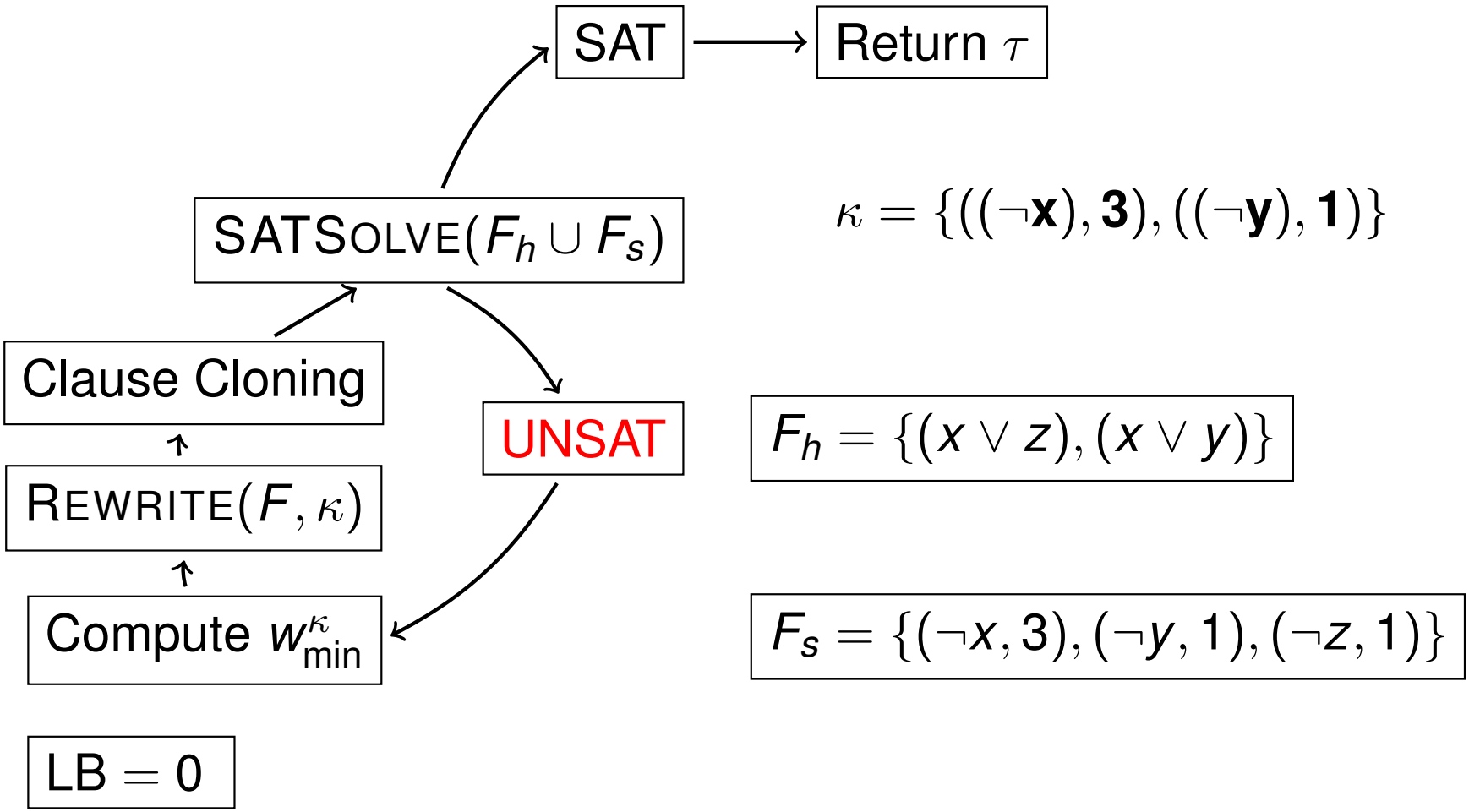
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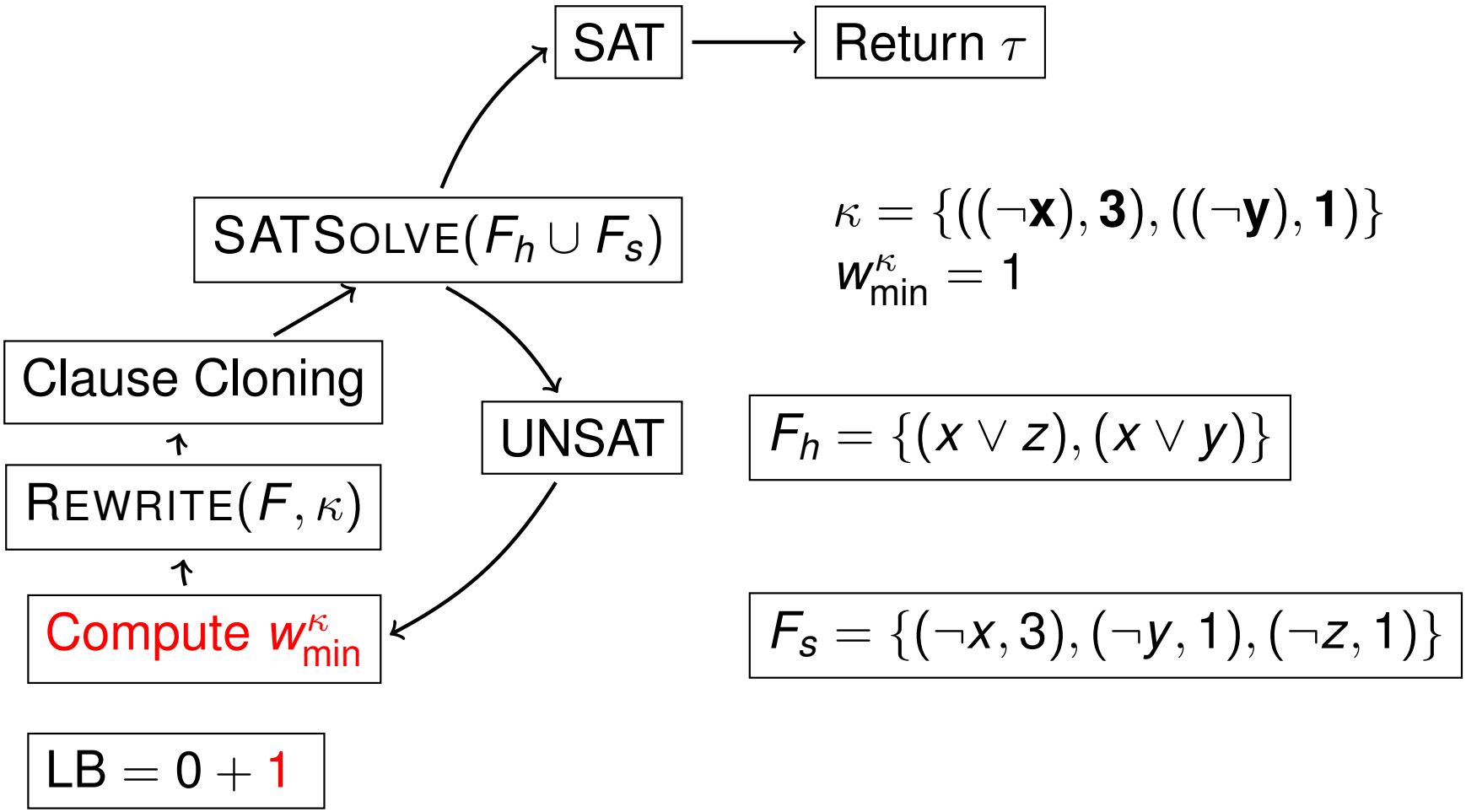
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LB $= 0 + 1$
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$w^{\kappa}_{\min} = 1$
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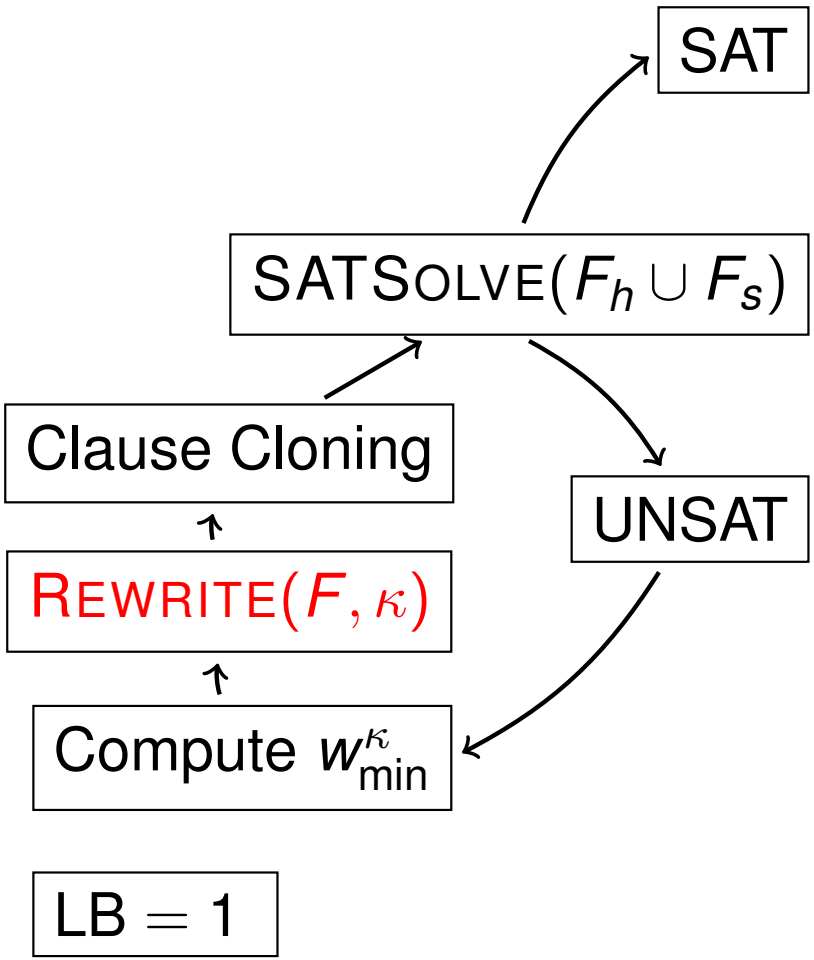

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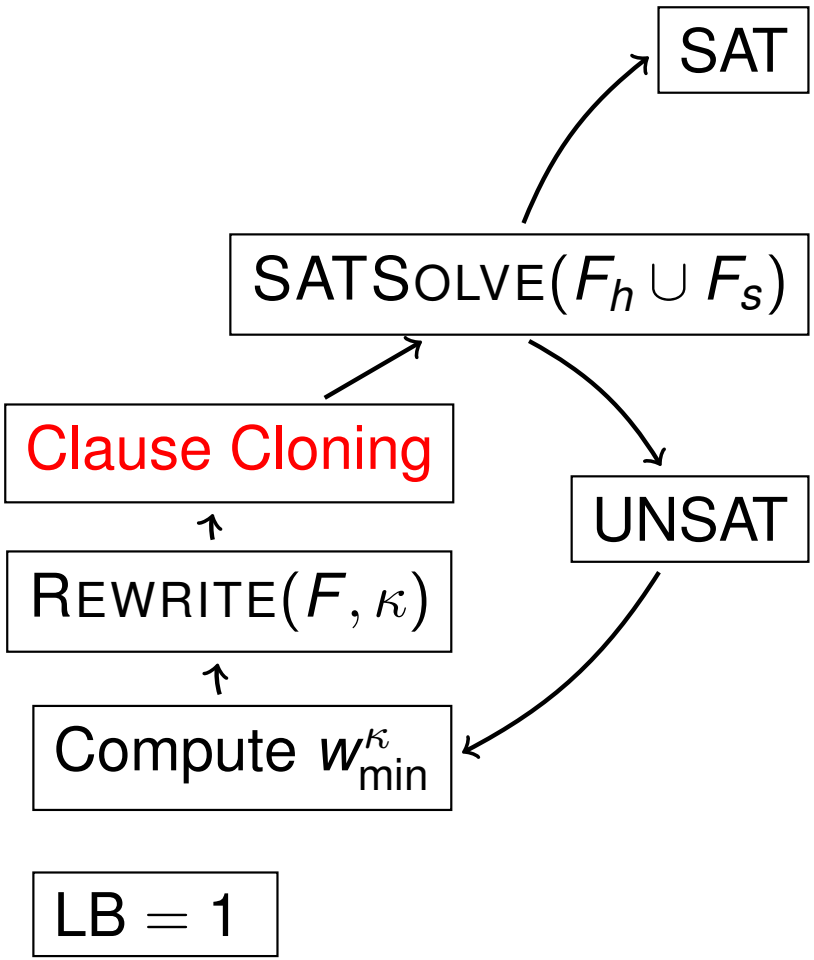

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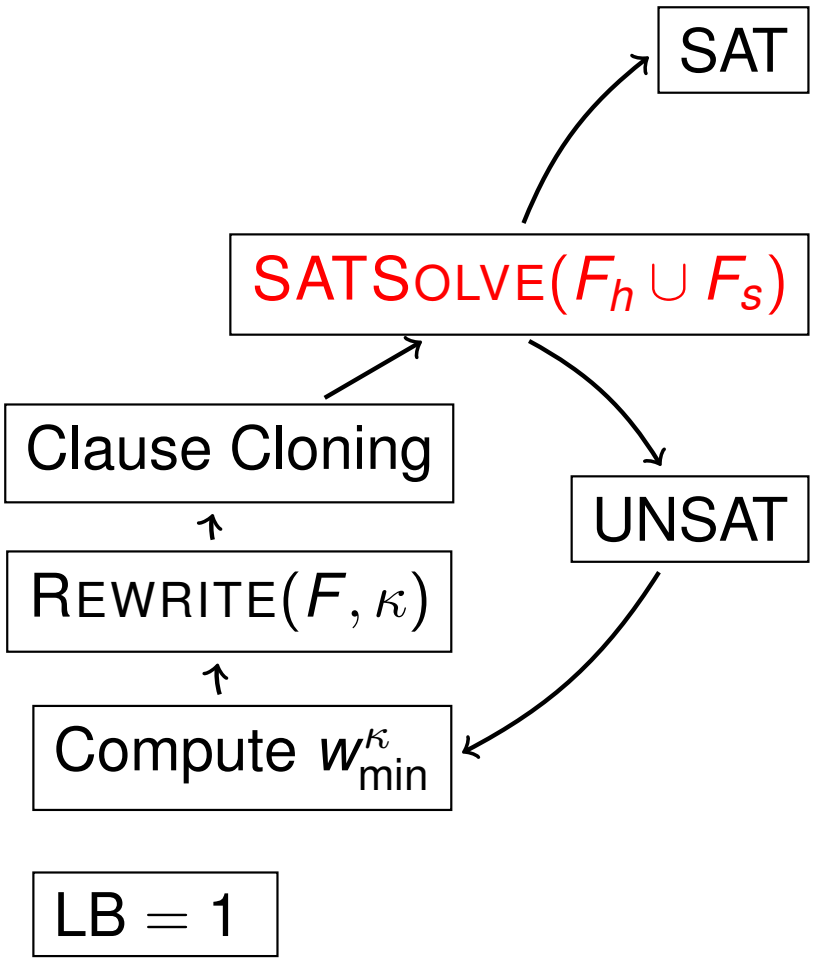

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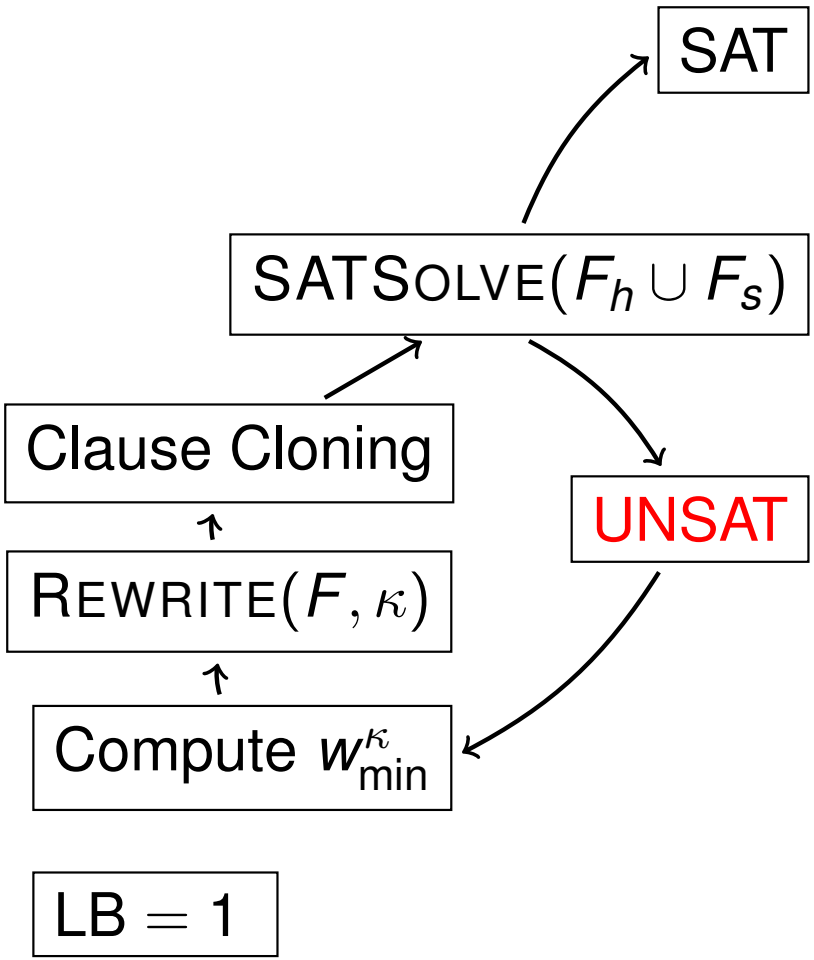

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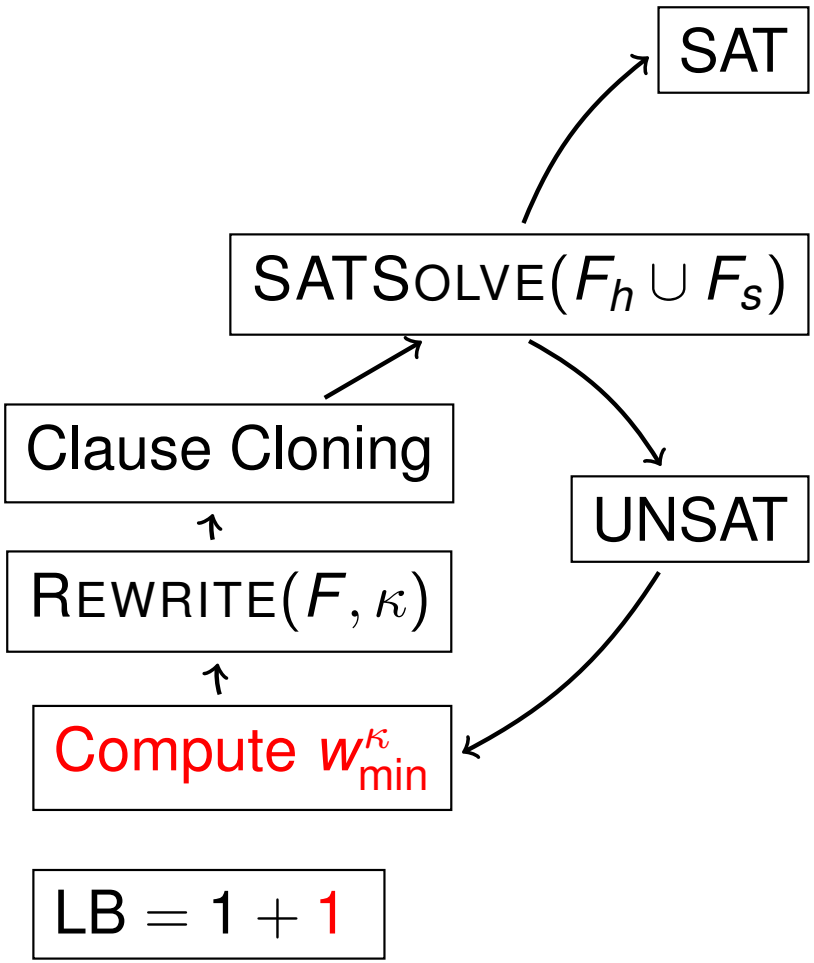

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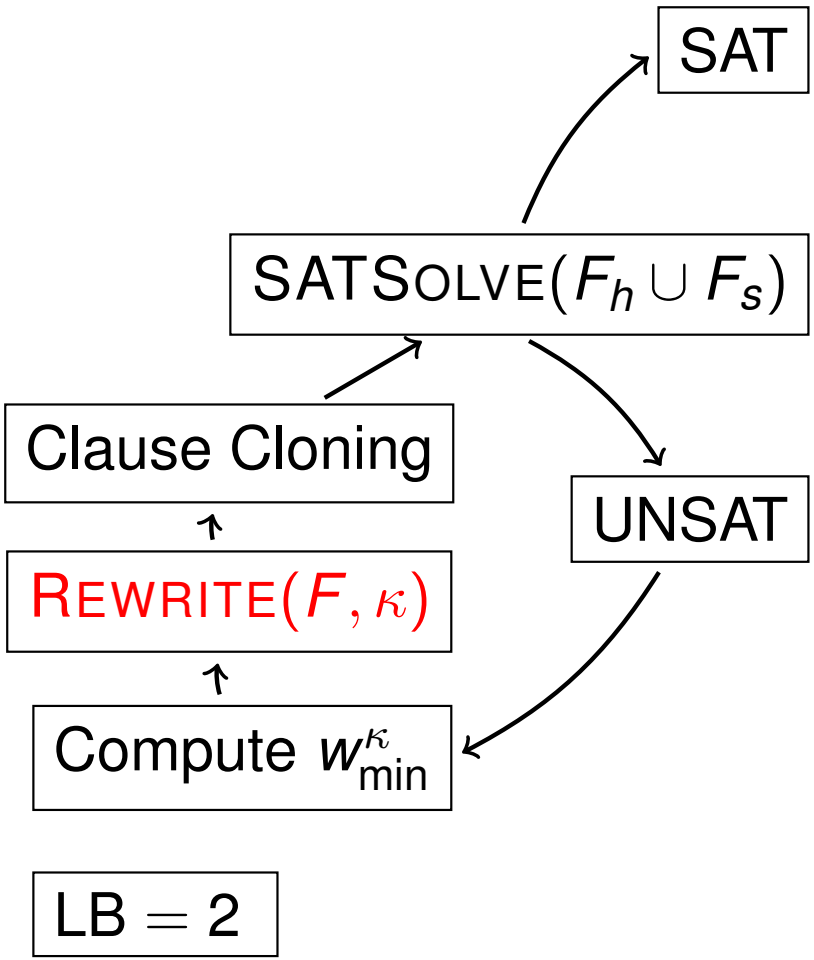

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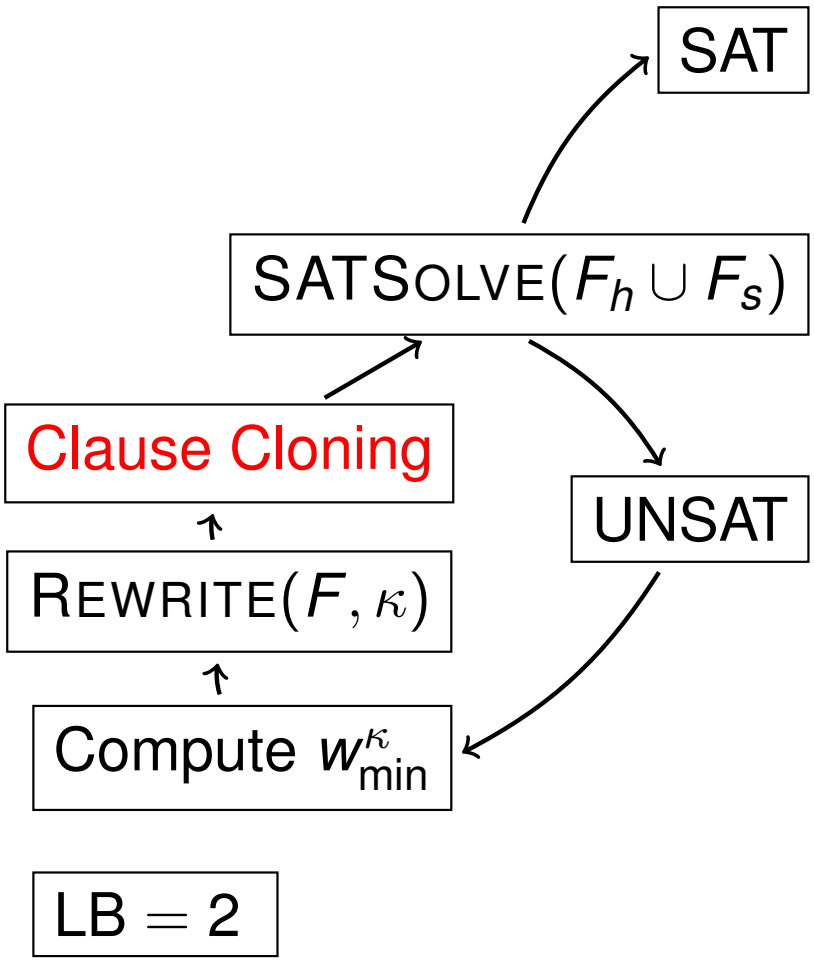

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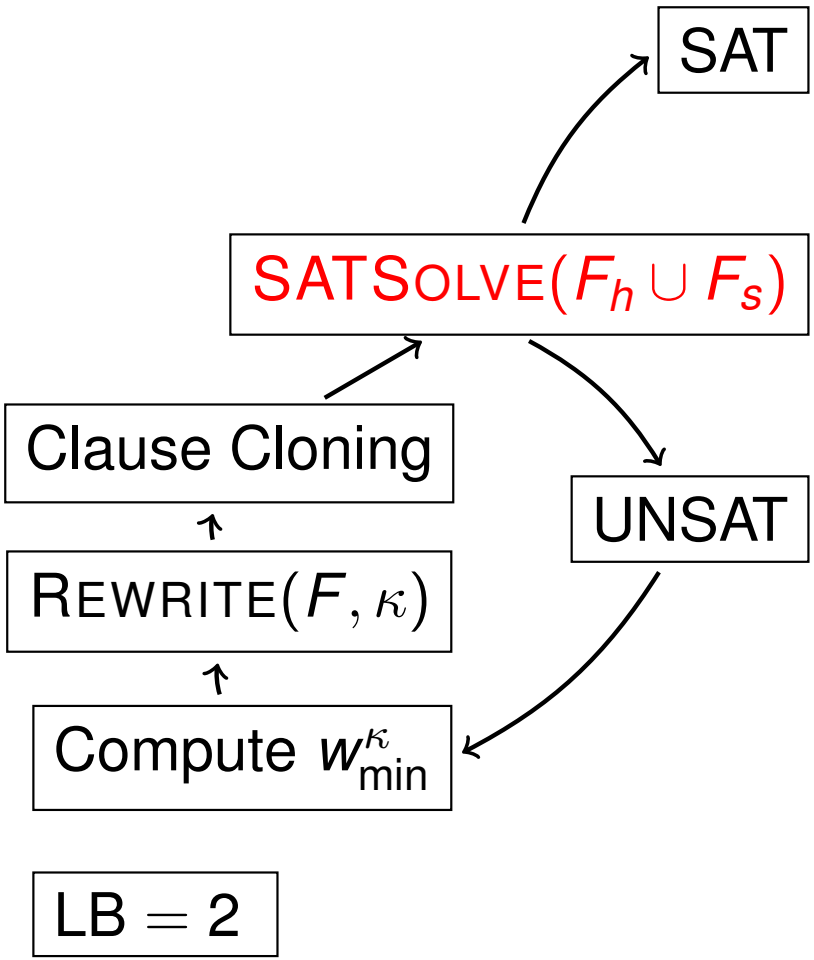

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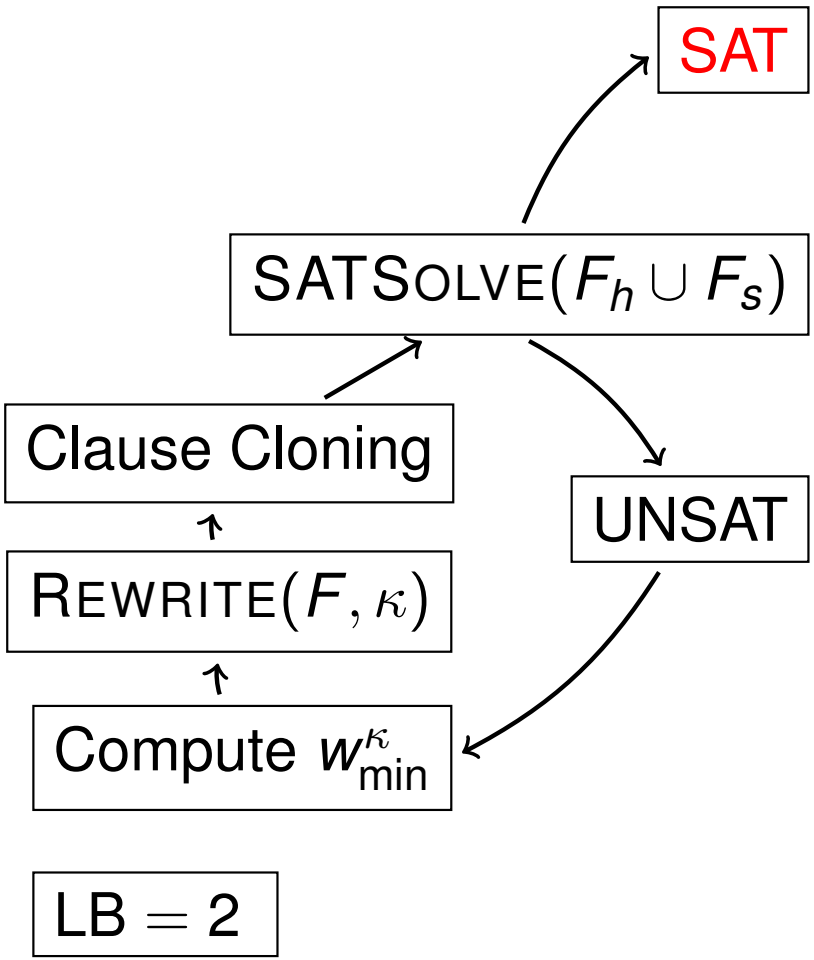

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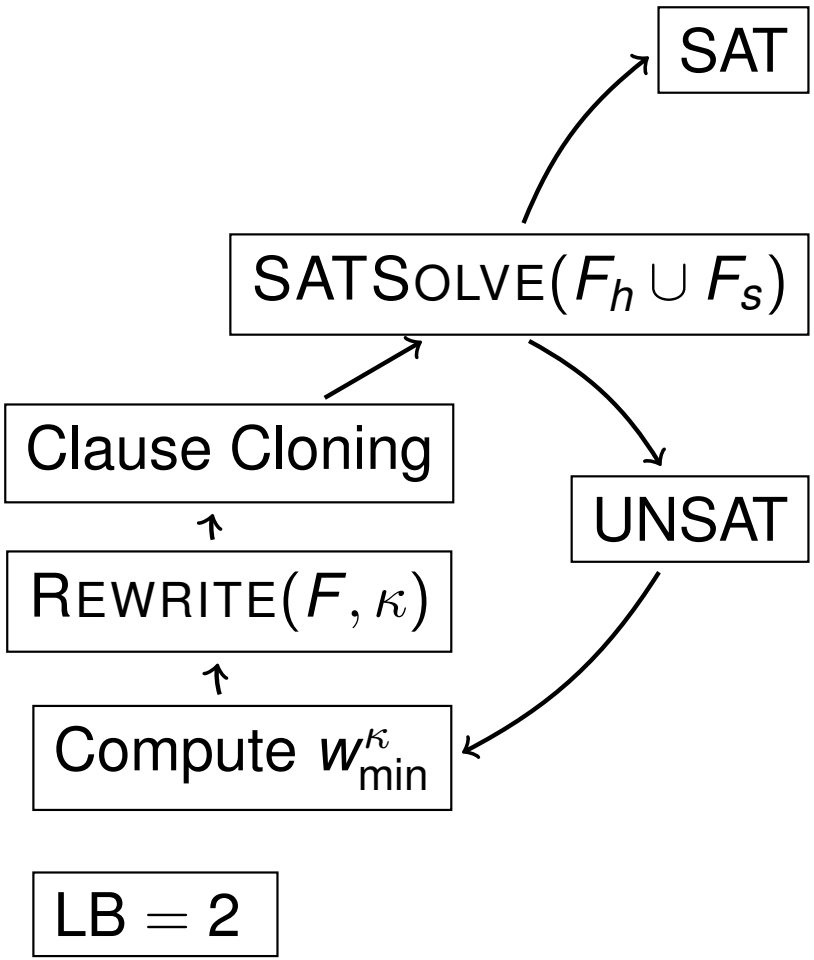

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Observations

- Due to clause weights, both cores in previous example need to be extracted.
- Both can be extracted without rewriting.
- Adding relaxation variables to clause clones introduces implicit equivalences
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Previous Example, κ valid without rewriting

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Equivalence between relaxation variables

$F_h = \{(x \vee z), (x \vee y), (\neg x \vee r_1), (\neg y \vee r_2), (\neg x \vee r_3), (\neg z \vee r_4)\}$
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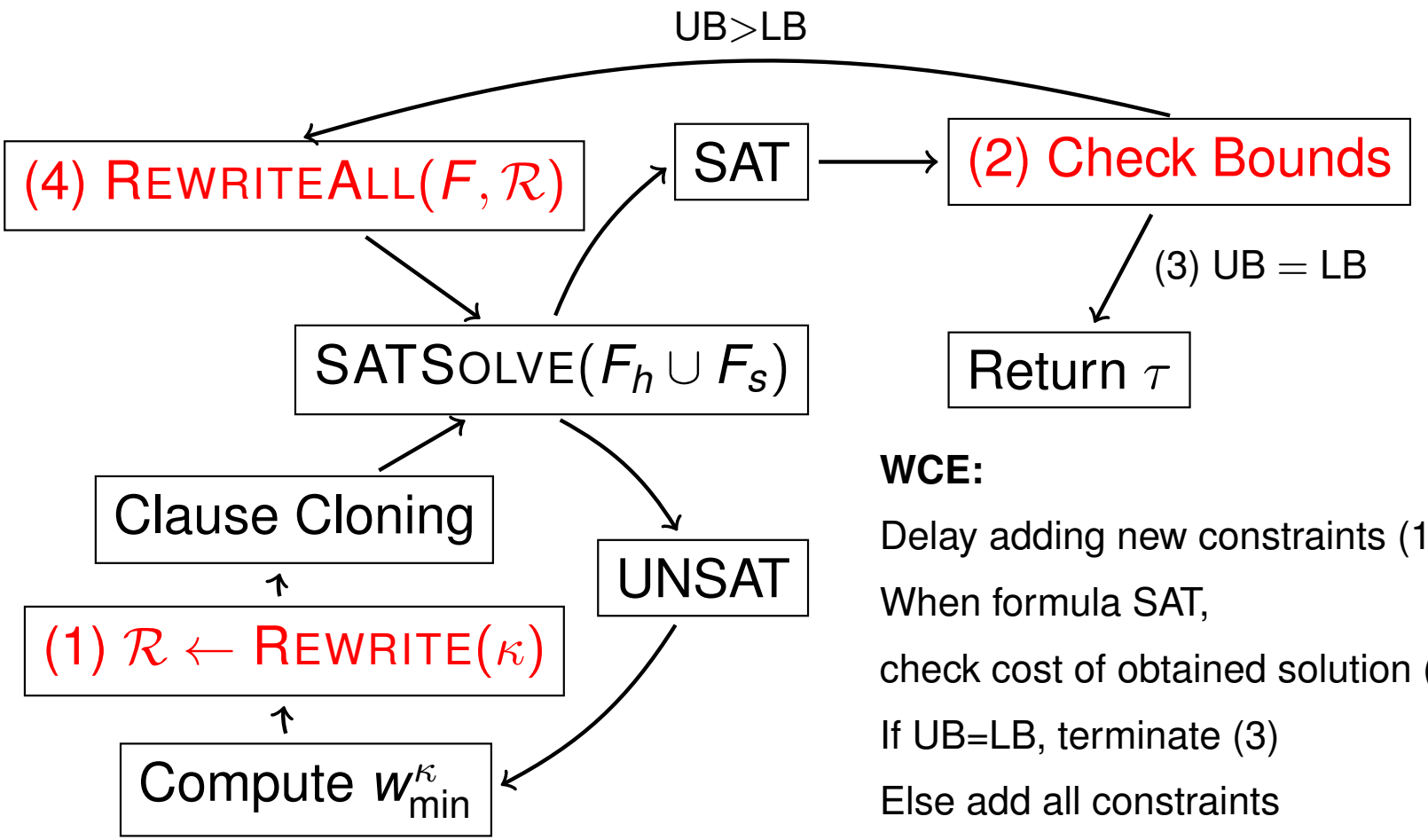
Weight Aware Core Extraction

Weight Aware Core Extraction (WCE)

- Delay relaxation steps, extract more valid cores.
- Easier on the SAT solver.
- Generic technique.
- Resembles earlier work on computing lower bounds Heras et al. [2012]

Core Guided weighted MaxSAT with WCE



WCE:

Delay adding new constraints (1)

When formula SAT,

check cost of obtained solution (UB) (2)

If UB=LB, terminate (3)

Else add all constraints

to formula (4)

Example with WCE

$$F_h = \{(x \vee z), (x \vee y)\}$$
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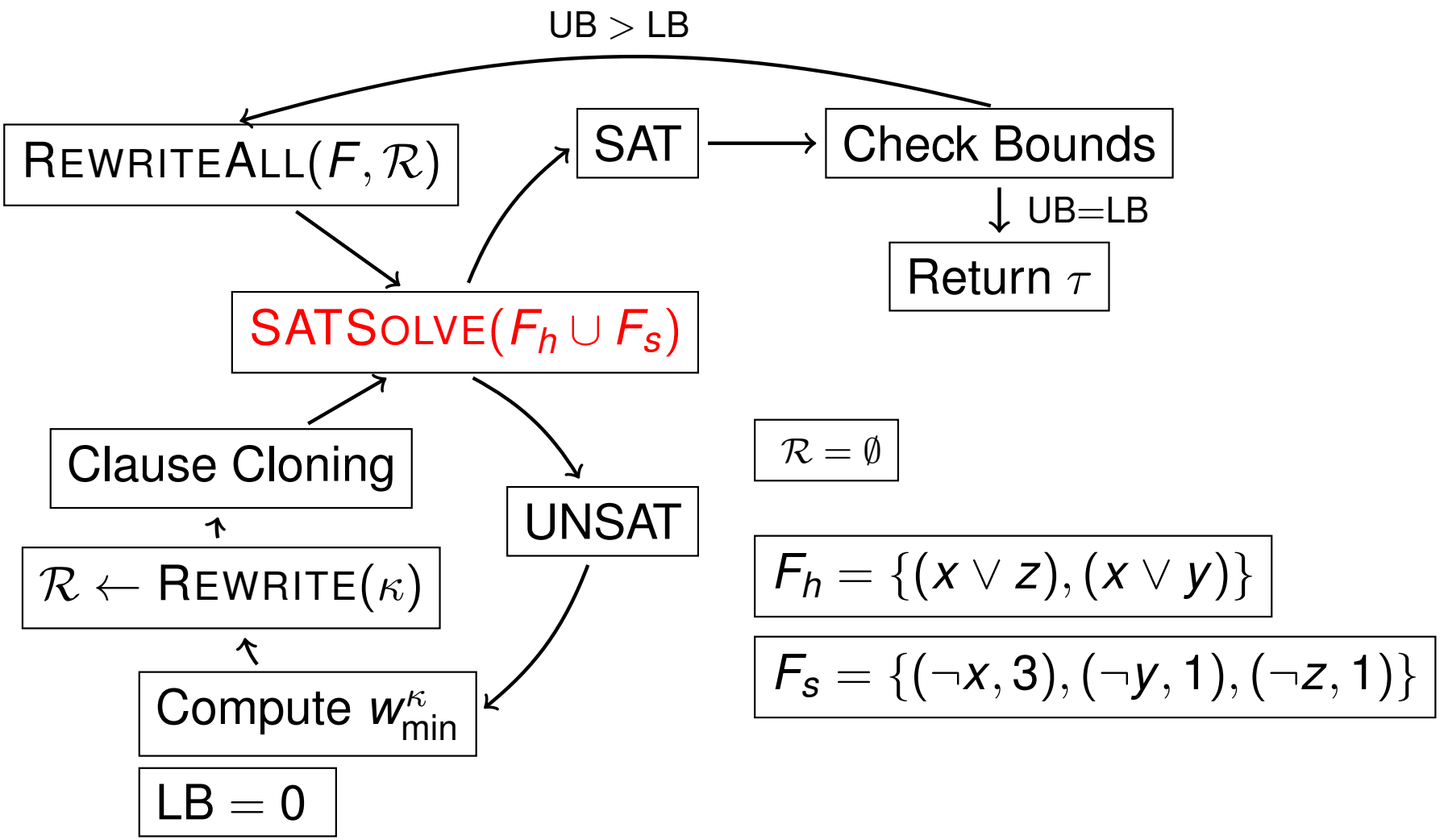

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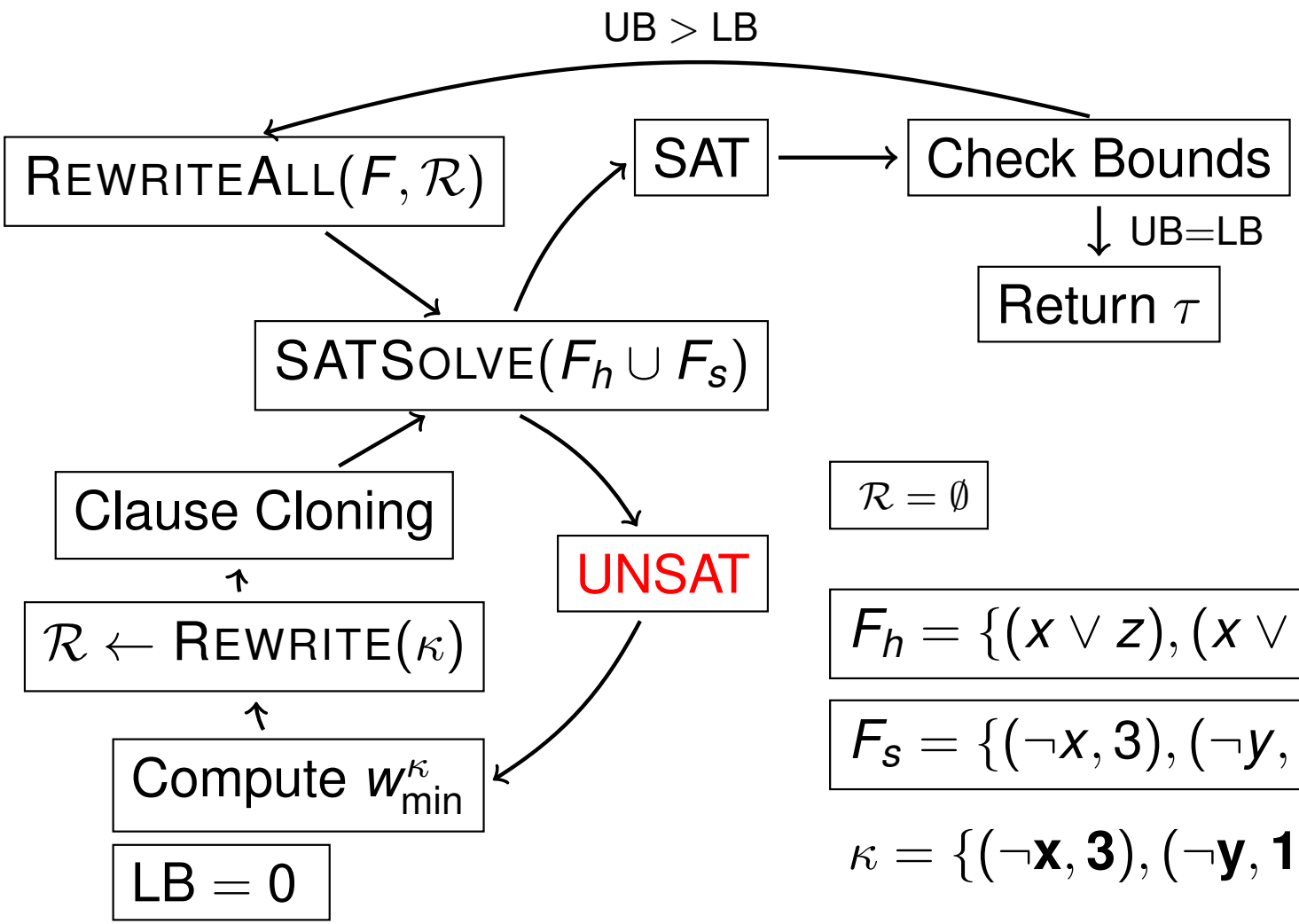


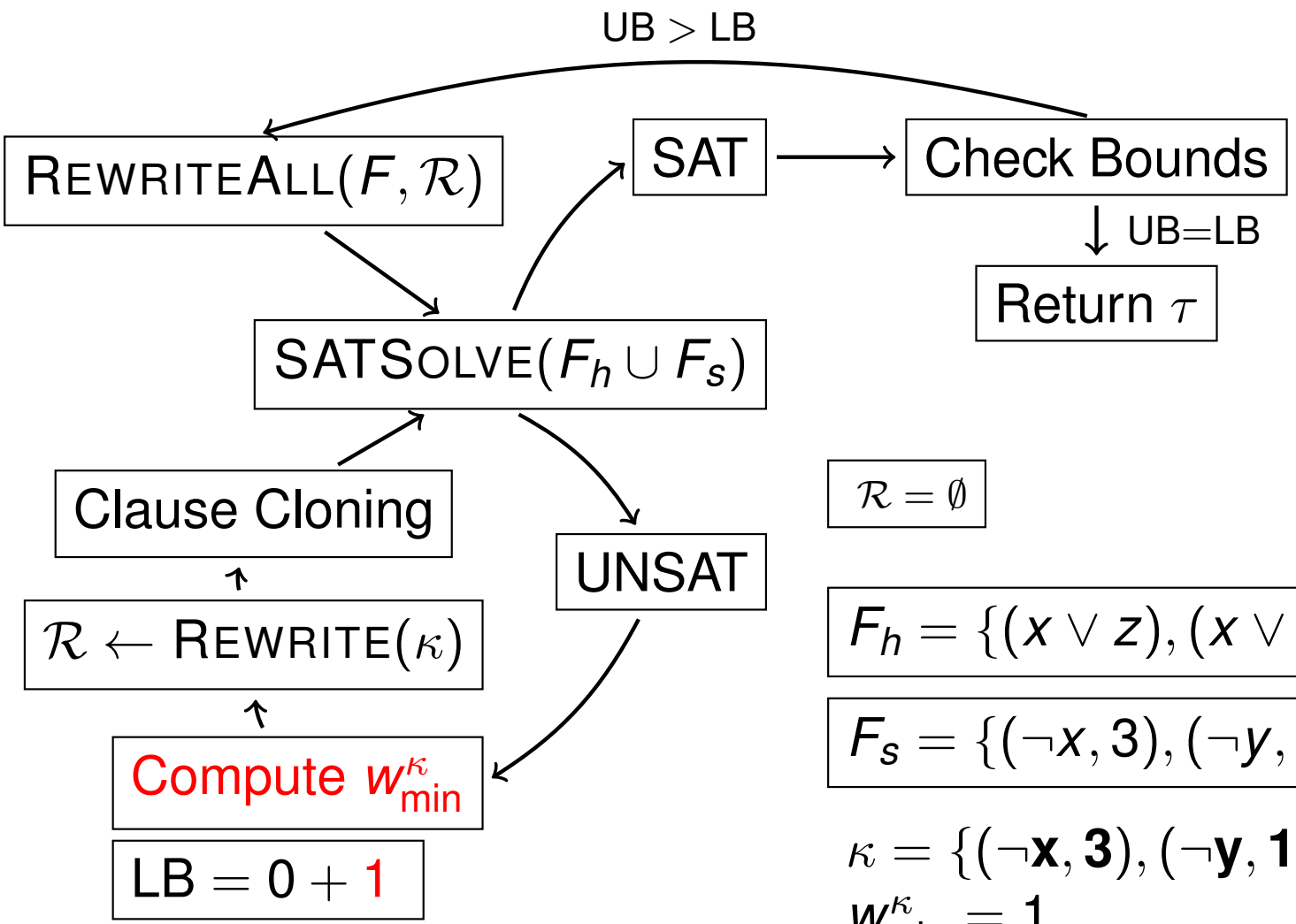
$\mathcal{R} = \emptyset$

$F_h = \{(x \vee z), (x \vee y)\}$

$F_s = \{(\neg x, 3), (\neg y, 1), (\neg z, 1)\}$

$\kappa = \{(\neg \mathbf{x}, \mathbf{3}), (\neg \mathbf{y}, \mathbf{1})\}$

Example with WCE



$\mathcal{R} = \emptyset$

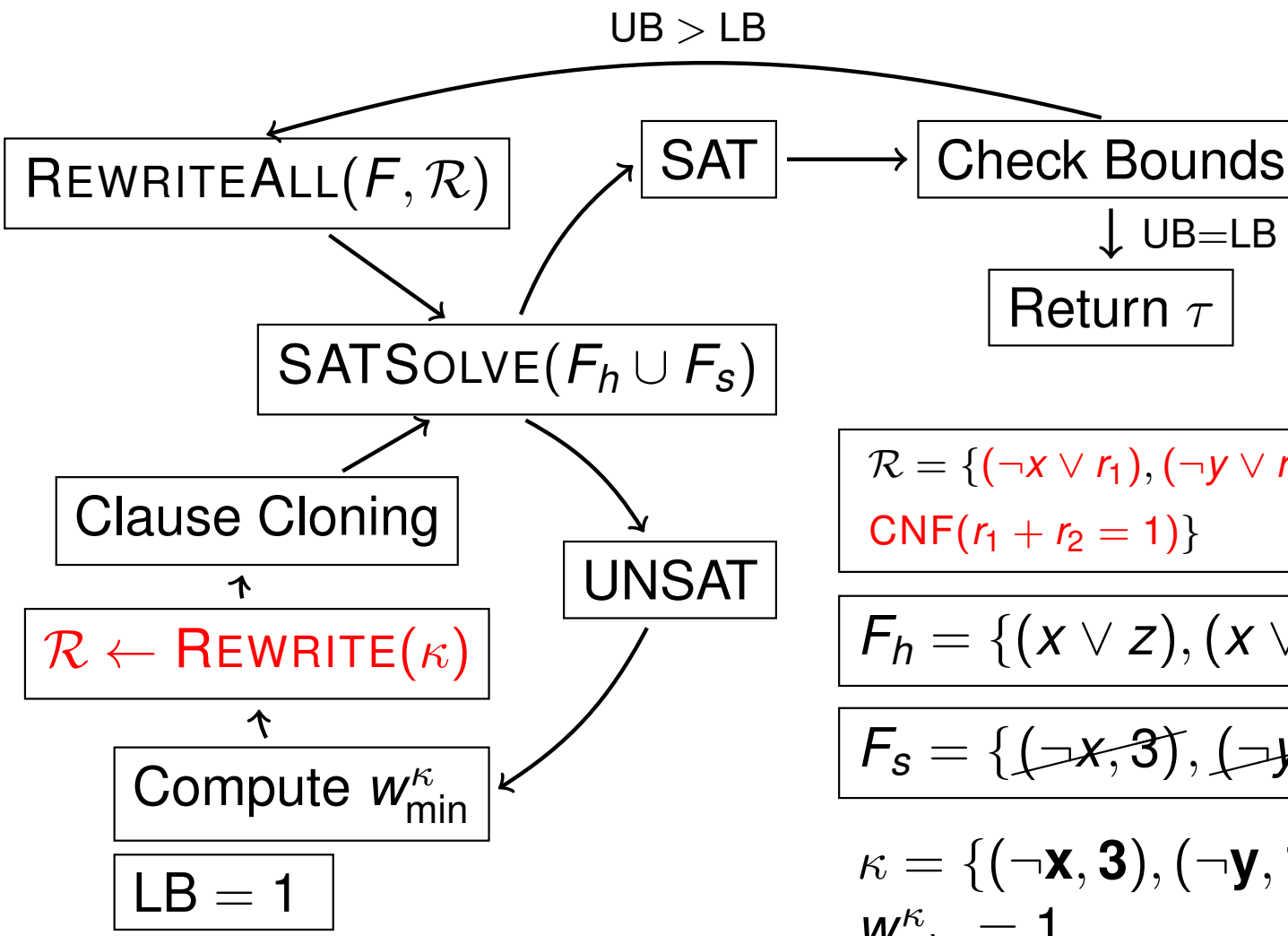
$F_h = \{(x \vee z), (x \vee y)\}$

$F_s = \{(\neg x, 3), (\neg y, 1), (\neg z, 1)\}$

$\kappa = \{(\neg \mathbf{x}, \mathbf{3}), (\neg \mathbf{y}, \mathbf{1})\}$

$w^{\kappa}_{\text{min}} = 1$

Example with WCE



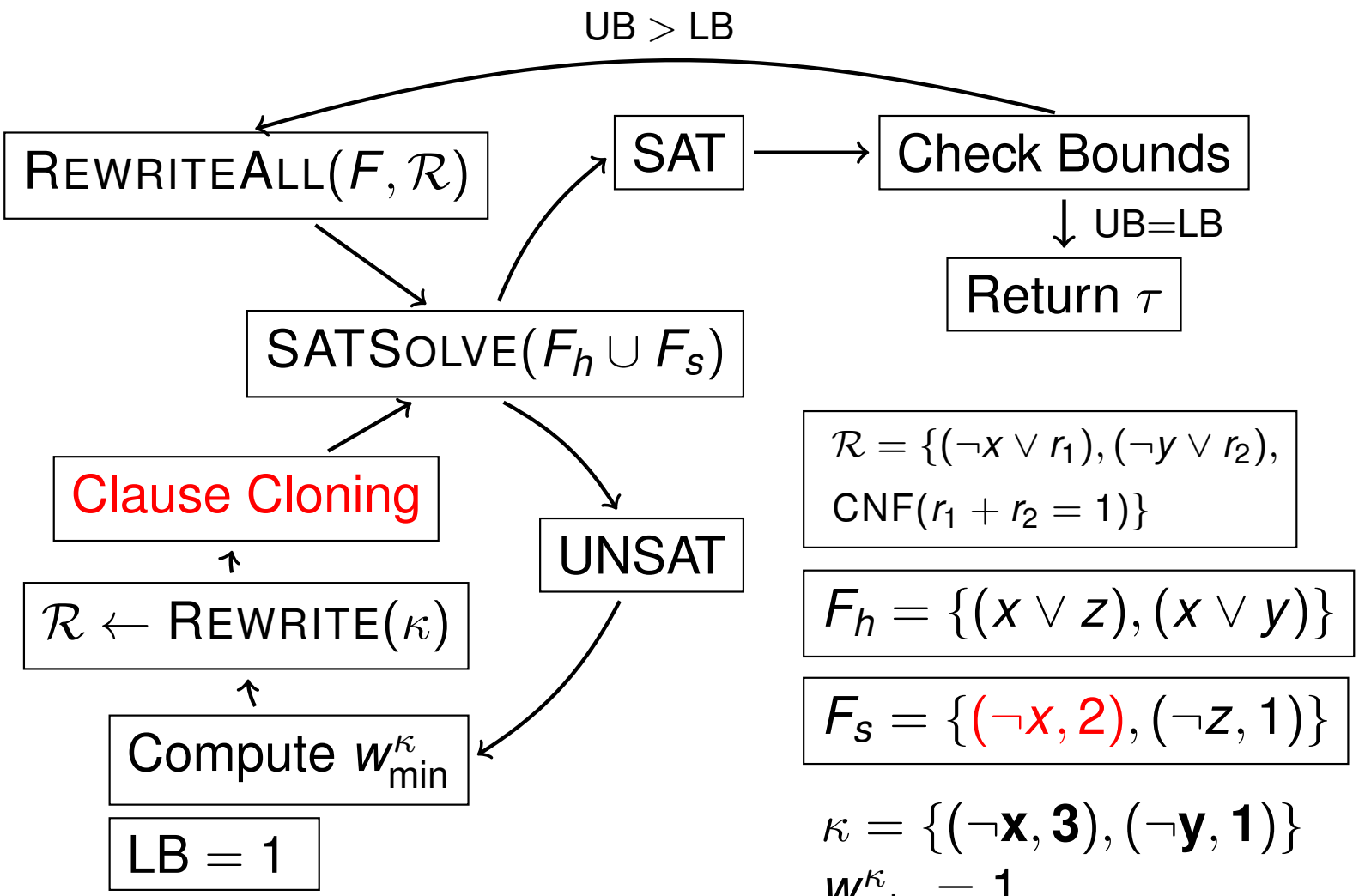
$\mathcal{R} = \{(\neg x \vee r_1), (\neg y \vee r_2),$
$\mathrm{CNF}(r_1 + r_2 = 1)\}$

$F_h = \{(x \vee z), (x \vee y)\}$

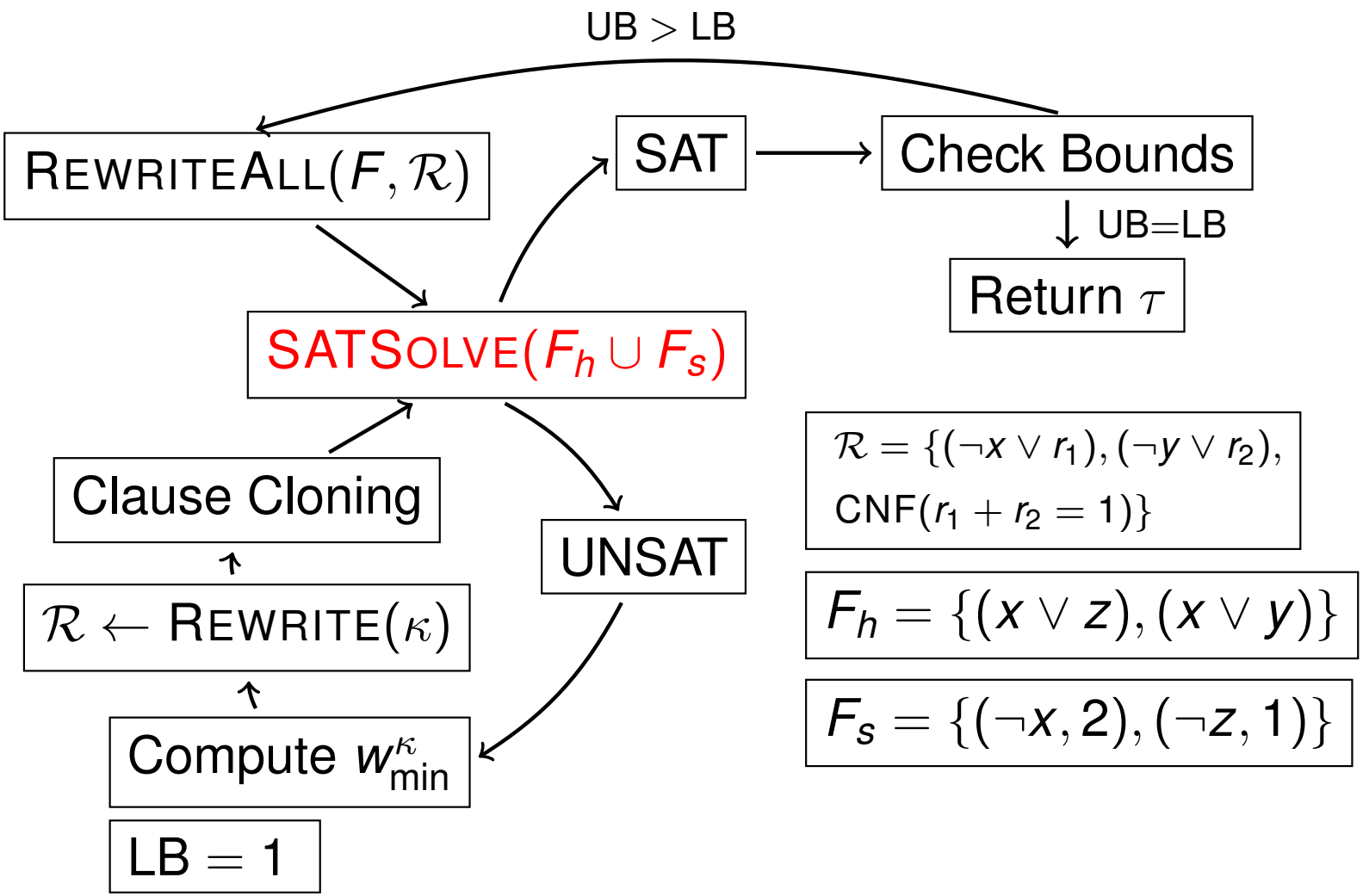
$F_s = \{\cancel{(\neg x, 3)}, \cancel{(\neg y, 1)}, (\neg z, 1)\}$

$\kappa = \{(\neg \mathbf{x}, \mathbf{3}), (\neg \mathbf{y}, \mathbf{1})\}$
$w^{\kappa}_{\min} = 1$

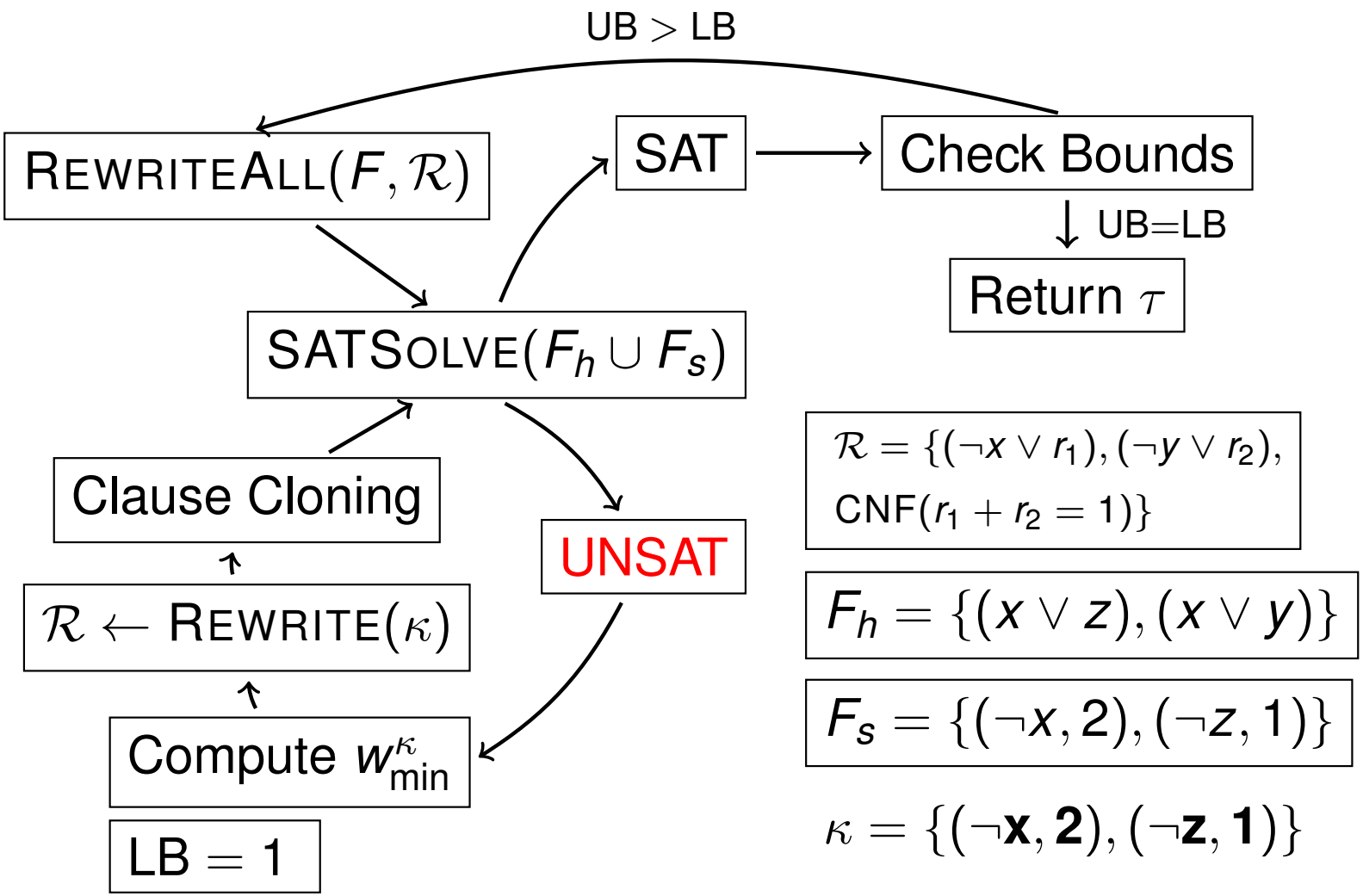
Example with WCE



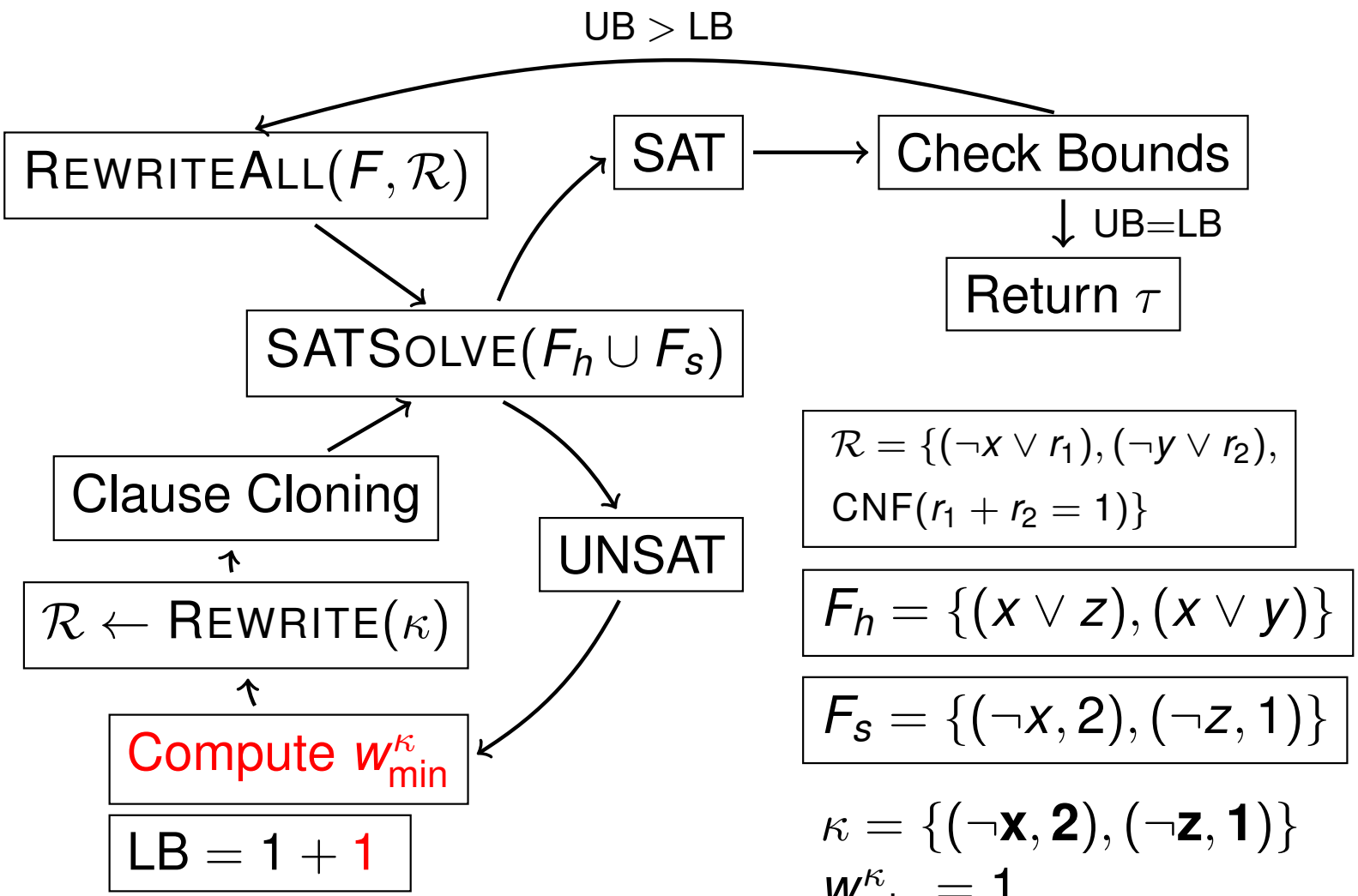
Example with WCE



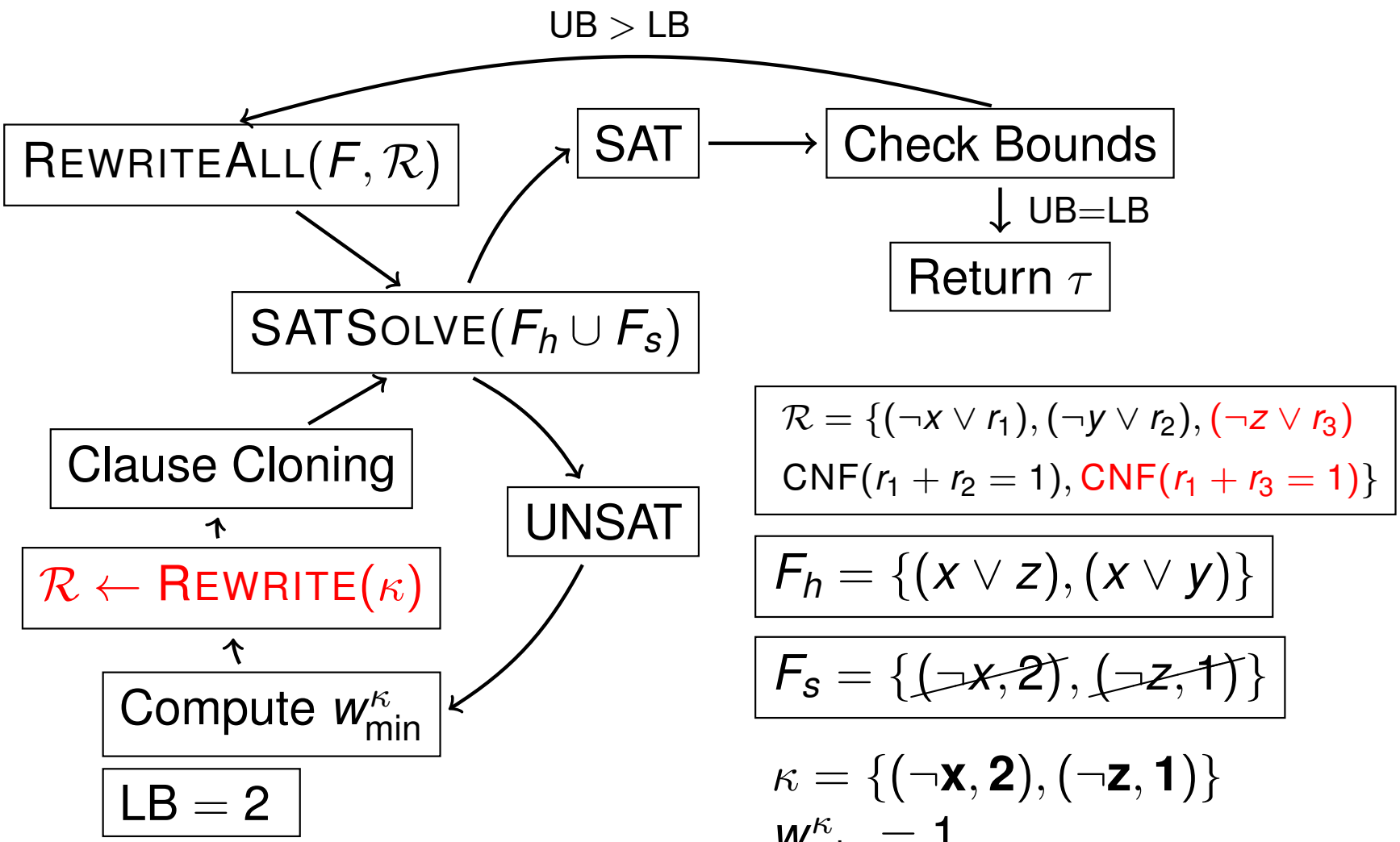
Example with WCE



Example with WCE



Example with WCE



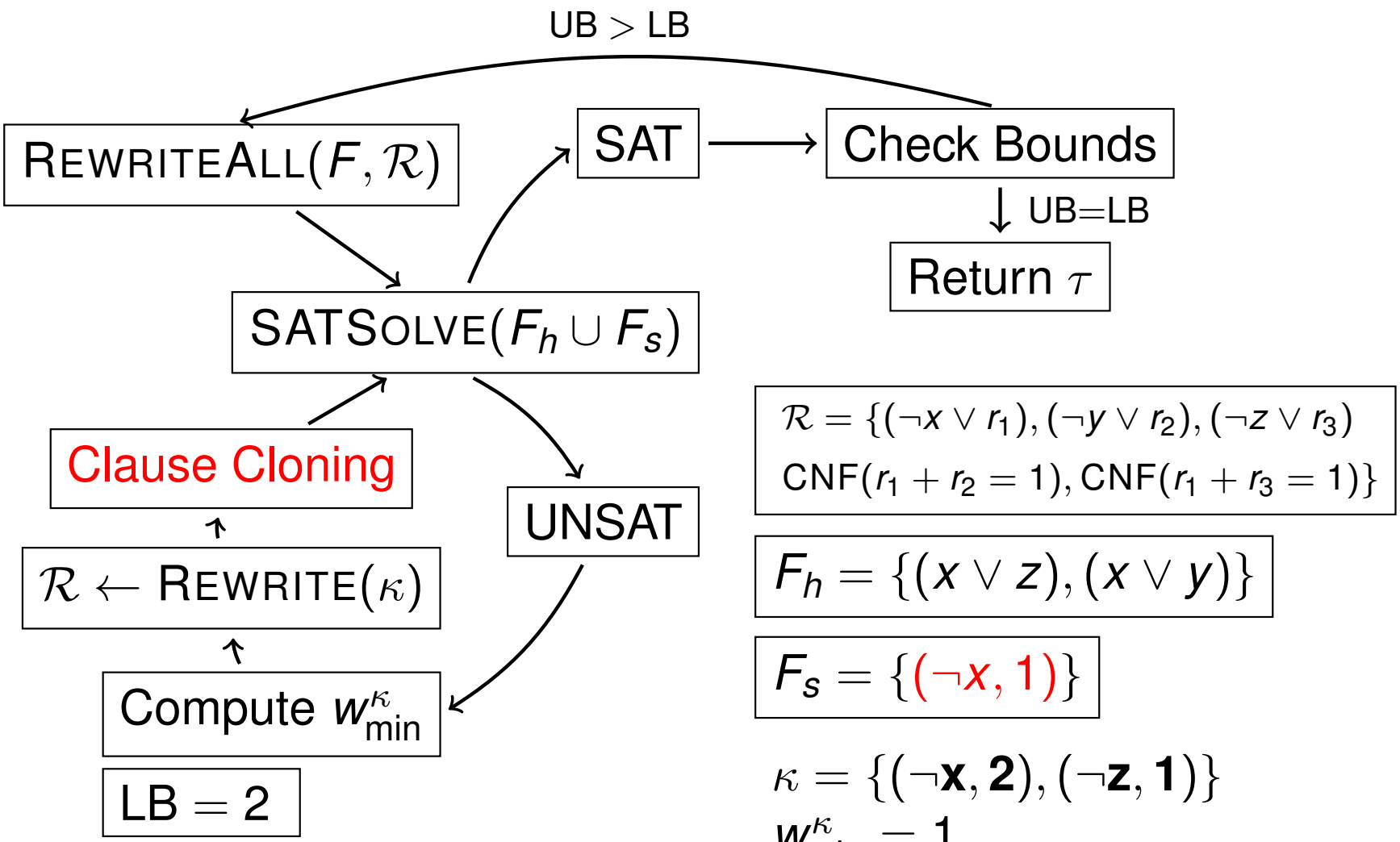
$\mathcal{R} = \{(\neg x \vee r_1), (\neg y \vee r_2), (\neg z \vee r_3)$
$\text{CNF}(r_1 + r_2 = 1), \text{CNF}(r_1 + r_3 = 1)\}$

$F_h = \{(x \vee z), (x \vee y)\}$

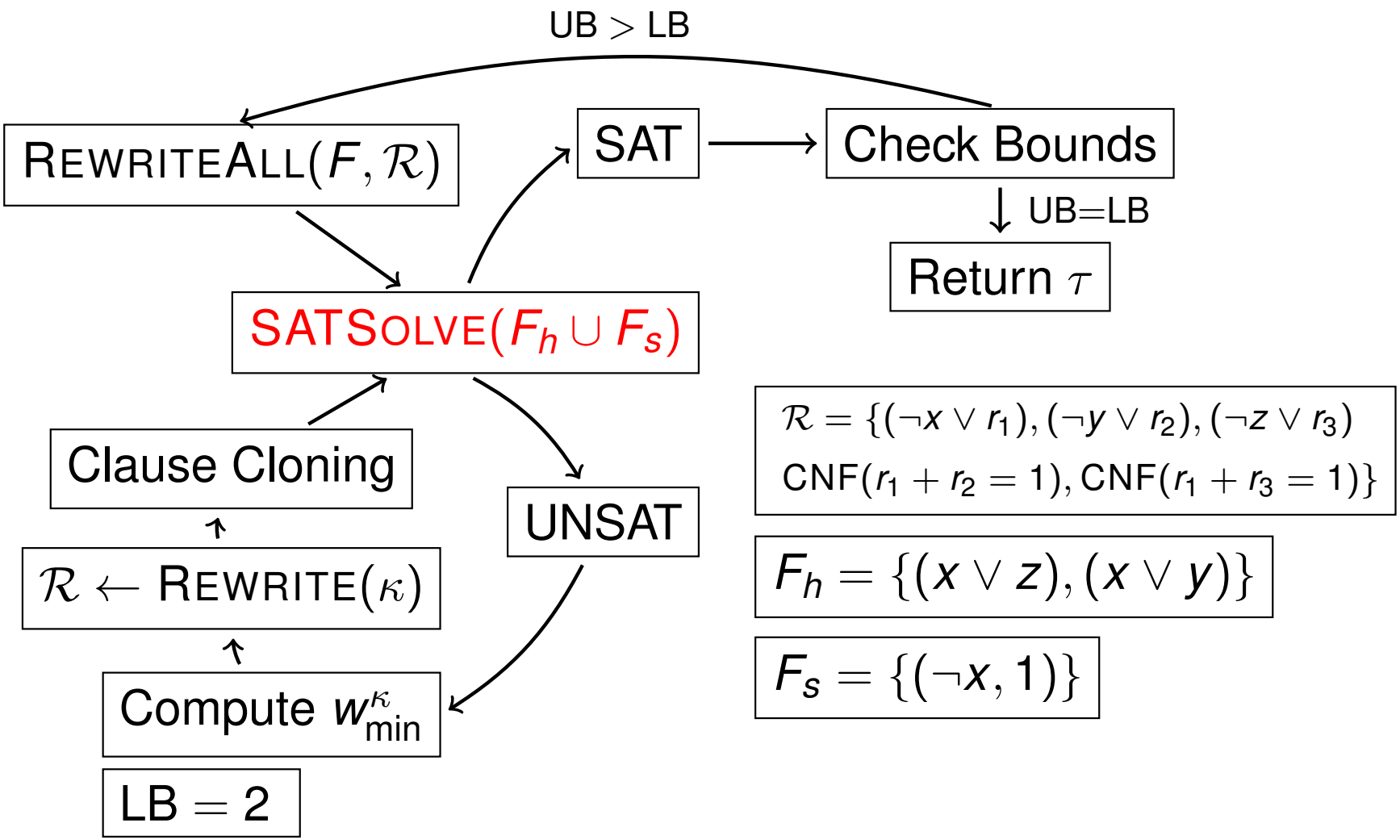
$F_s = \{\cancel{(\neg x, 2)}, \cancel{(\neg z, 1)}\}$

$\kappa = \{(\neg \mathbf{x}, \mathbf{2}), (\neg \mathbf{z}, \mathbf{1})\}$
$w^{\kappa}_{\text{min}} = 1$

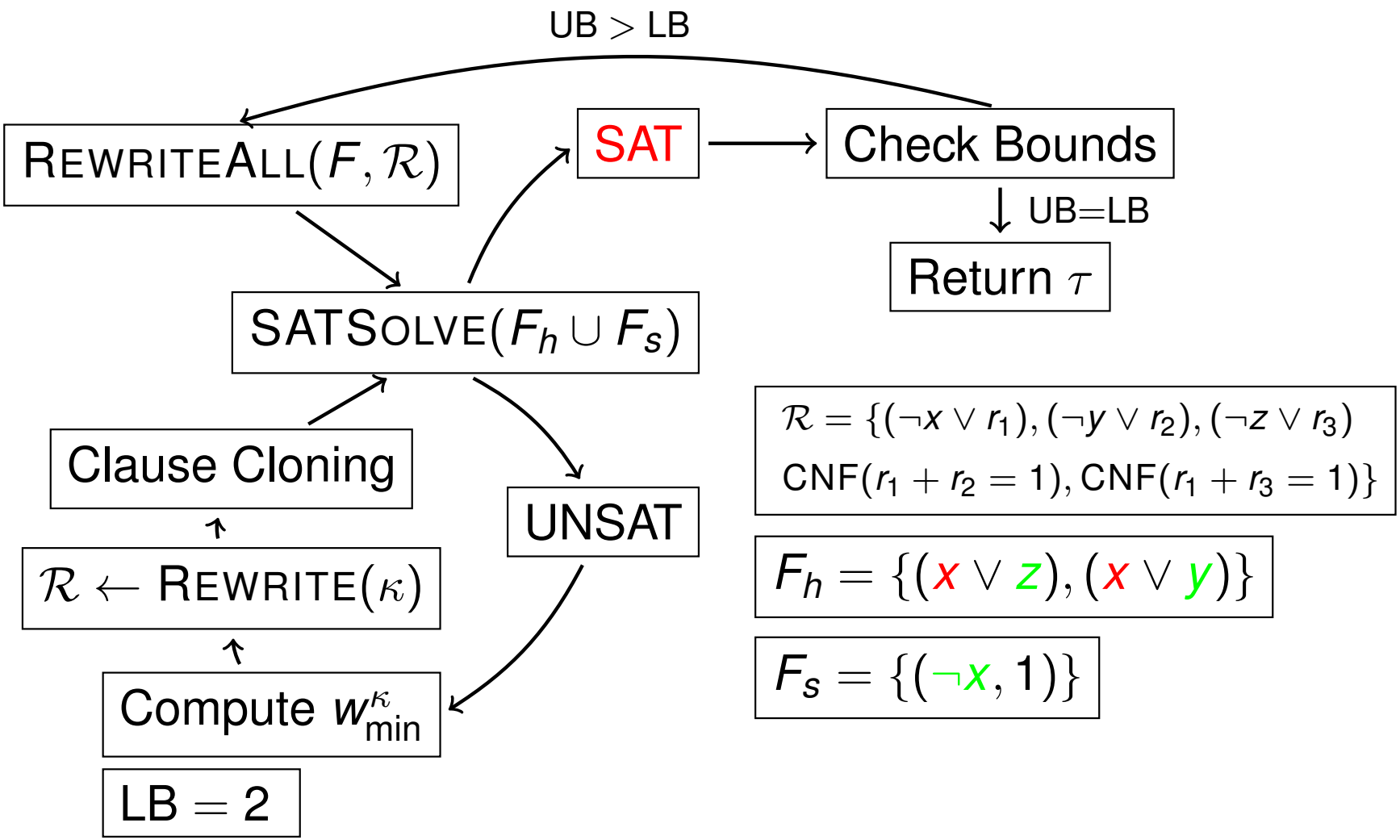
Example with WCE



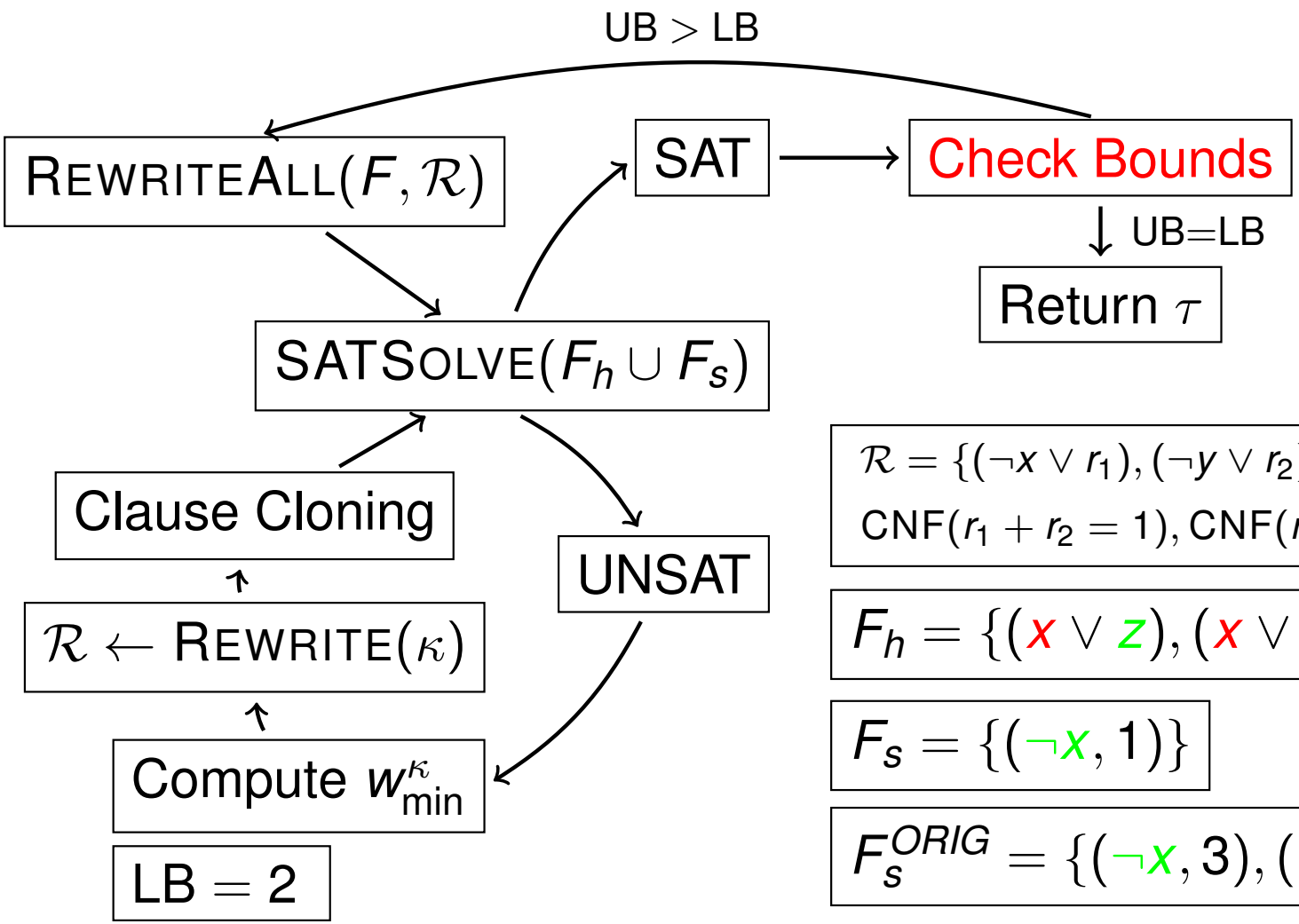
Example with WCE



Example with WCE



Example with WCE



$\mathcal{R} = \{(\neg x \vee r_1), (\neg y \vee r_2), (\neg z \vee r_3)$
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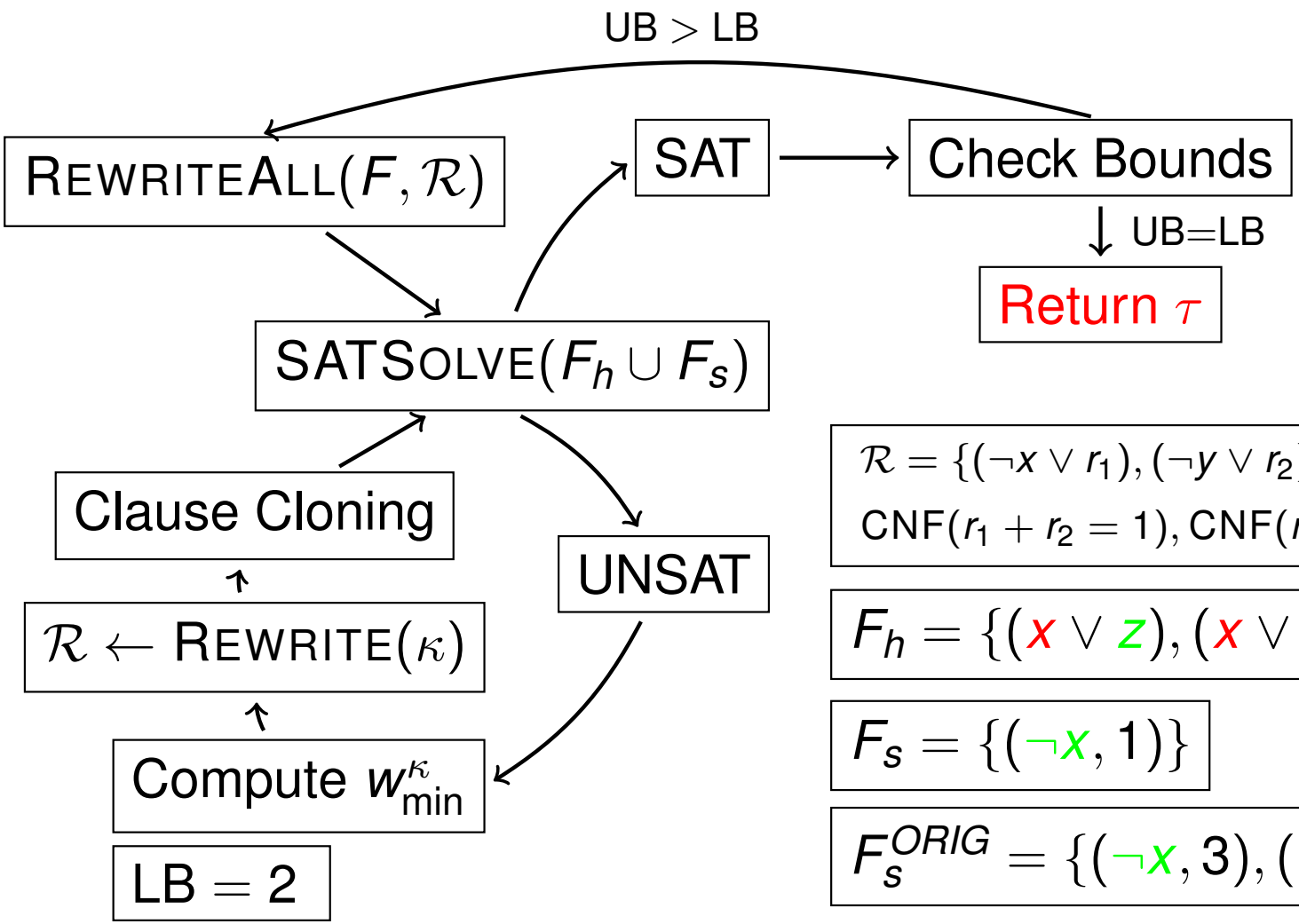
$F_h = \{(x \vee z), (x \vee y)\}$

$F_s = \{(\neg x, 1)\}$

$F_s^{ORIG} = \{(\neg x, 3), (\neg y, 1), (\neg z, 1)\}$

UB=2

Example with WCE



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Implementing WCE

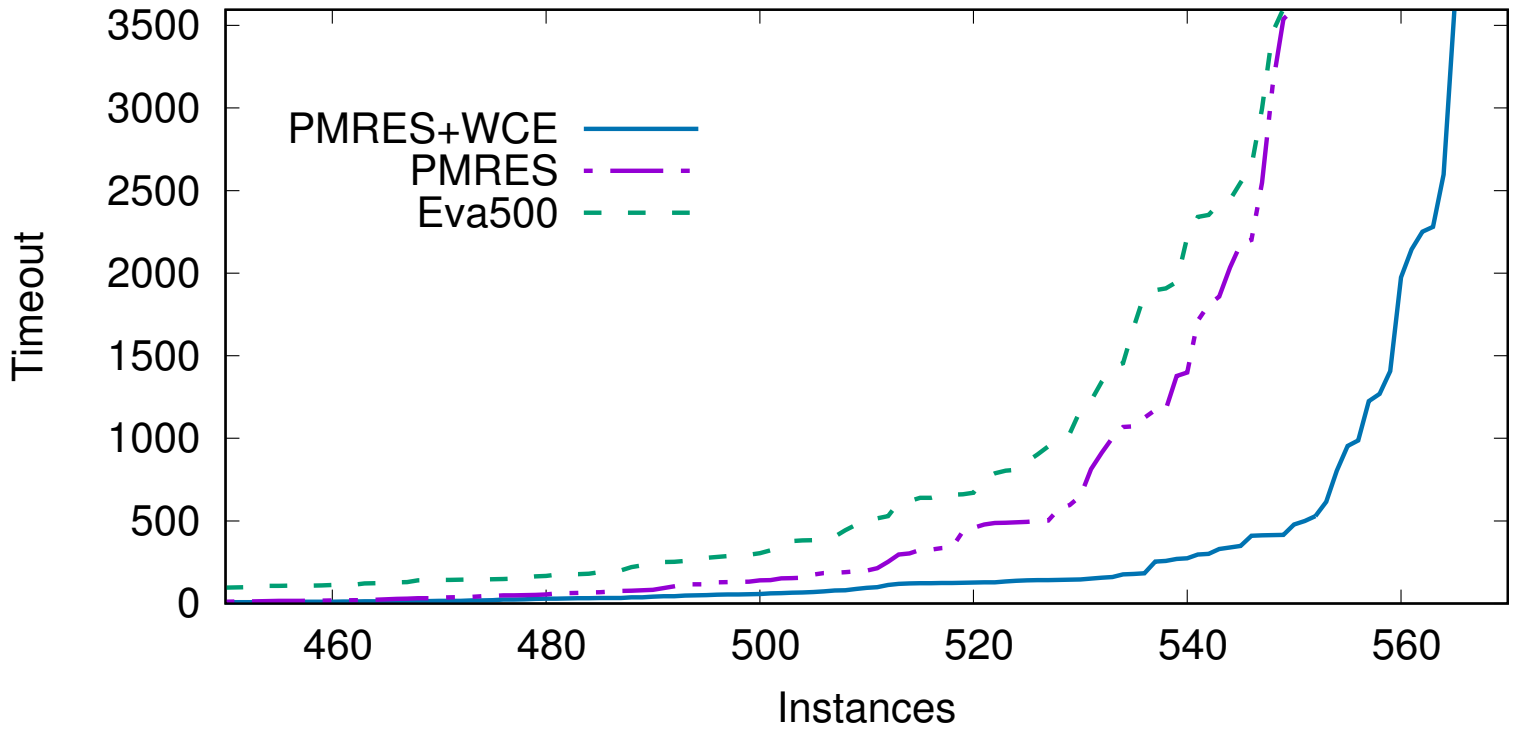
- Simple observation: a hardened clause $C \vee r$ does not appear in future cores.
- Allows implementing clause cloning through assumption variables (and no actual cloning)
- Makes WCE simple to implement.
- PMRES, Primal-Dual, OLL, K, etc. Morgado et al. [2014]; Narodytska and Bacchus [2014]; Bjørner and Narodytska [2015]; Alviano et al. [2015].

Experimental Evaluation

Set up of Experiments

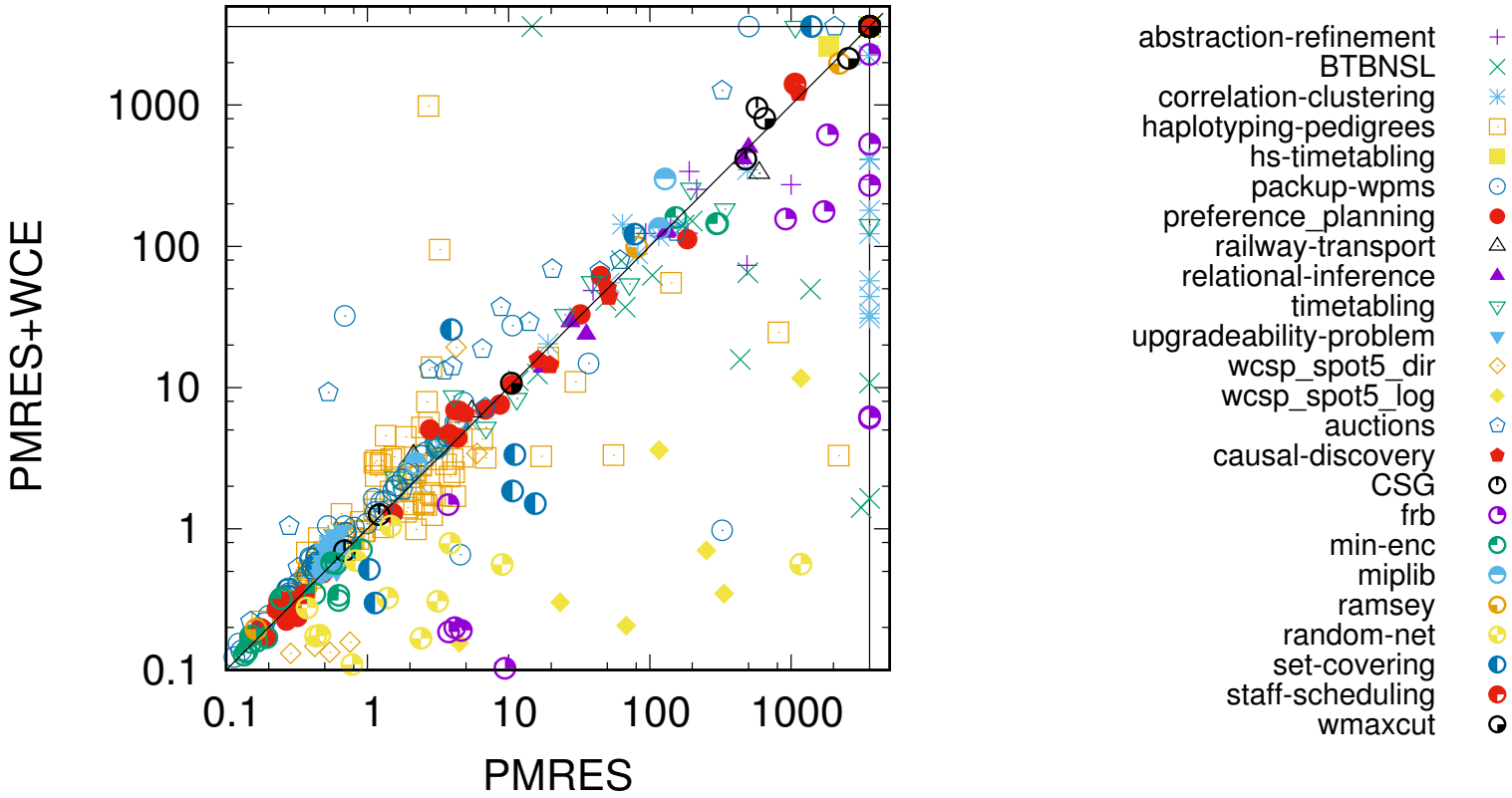
- Reimplemented the PMRES algorithm on top of Open-WBO
  Narodytska and Bacchus [2014]; Martins et al. [2014b]
- Compare to the original implementation, Eva500
- Benchmarks: MaxSAT Evaluation 2016 WPMS industrial (630) and crafted (321) instances.

Experimental Evaluation



Number of instances solved (x-axis) by the individual solvers under different per-instance time limits (y-axis)

Experimental Evaluation



Per-instance running time comparison of PMRES (x-axis) and PMRES+WCE (y-axis)

Comparison to OLL Morgado et al. [2014]

Industrial Domain (#instances)	MSCG15b	PMRES+WCE
abstraction refinement (11)	10 (17096)	10 (**2147**)
BTBNSL (60)	13 (885)	**19** (**679**)
correlation clustering (129)	**33** (19780)	19 (4499)
haplotyping pedigrees (100)	100 (**1343**)	100 (1374)
hs-timetabling (14)	1 (**167**)	1 (2596)
packup-wpms (99)	**99** (410)	94 (191)
preference planning (29)	29 (2021)	29 (**264**)
railway transport (11)	3 (**283**)	3 (340)
relational inference (9)	4 (3167)	**8** (**1360**)
timetabling (26)	12 (**764**)	12 (878)
upgradeability (100)	100 (118)	100 (**59**)
wcsp_spot5_dir (21)	**17** (2776)	14 (24)
wcsp_spot5_log (21)	14 (19)	14 (**17**)

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Summary

- Core Guided MaxSAT shown to be effective on industrial MaxSAT instances.
- Most Core Guided MaxSAT algorithms deal with weights through clause cloning
- We propose WCE, a refinement to clause cloning.
- Prove correctness and discus applicability of WCE on modern MaxSAT algorithms
- Experimentally show improvement on PMRES

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