Improving the Effectiveness of SAT-Based Preprocessing for MaxSAT

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Motivation

- **(Weighted-Partial) Maximum Satisfiability:** An NP-hard optimization paradigm based on the Boolean Satisfiability problem.
- Example instance:
  $F_h = \{(x \vee z), (\neg z), (y \vee z)\}$
  $F_s = \{(\neg x), (\neg y \vee \neg z), (z \vee y), (\neg z \vee y)\}$
- Find assignment τ that satisfies all hard and a maximum number of soft clauses.
  - Optimize over the weights on soft clauses.
- MaxSAT solving an active area of research.
- Solvers applied for cost-optimal solutions in various domains.

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Motivation

- **(Weighted-Partial) Maximum Satisfiability:** An NP-hard optimization paradigm based on the Boolean Satisfiability problem.
- Example instance:
  $F_h = \{(x \lor z), (\neg z), (y \lor z)\}$
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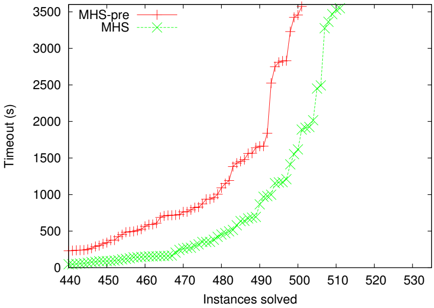
SAT-based preprocessing for MaxSAT

- *Preprocessing:* central technique in modern SAT solvers.
  Eén and Biere [2005]; Heule et al. [2010]; Järvisalo et al. [2012]
- Preprocessing MaxSAT has received only little attention.
  - Can be done using the so called Labelled-CNF (LCNF) framework
    Belov and Marques-Silva [2012]
  - Benefits still unclear
  - Can degrade performance of the solver.

SAT-based preprocessing for MaxSAT

- Example: The MaxHS algorithm. Davies and Bacchus [2013]
  One of the best performing solvers of the 2014 MaxSAT evaluation weighted partial crafted category.

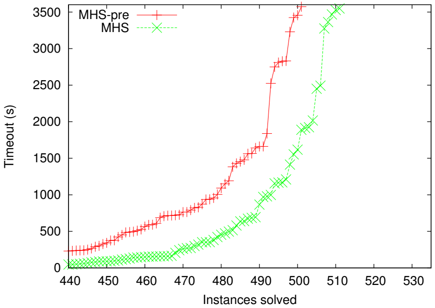


This Work

Focus on improving MaxHS by a tight integration of SAT-based preprocessing.

Main contributions

- Lifting of MaxHS to LCNFs.
- Tighter integration between MaxSAT preprocessing and solving.
- Improves solving time in practice.



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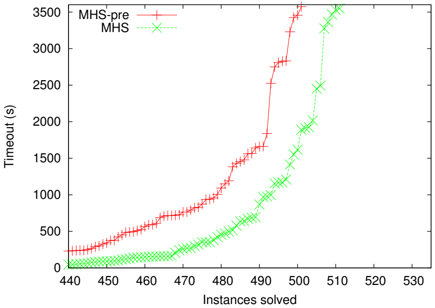

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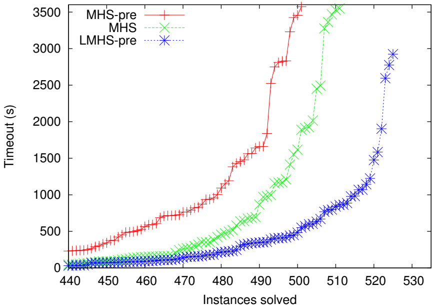


Direct application of MaxSAT preprocessing

	MaxSAT	**LCNF**
Soft Clauses	$(x \lor \neg z \lor \neg v)$	
	$(z \lor y)$	
	$(v \lor t)$	
	(t)	
Hard Clauses	$(y \lor t \lor \neg x)$	

Direct application of MaxSAT preprocessing

Conversion to LCNF adds labels to all soft clauses

	MaxSAT		LCNF
Soft Clauses	$(x \lor \neg z \lor \neg v)$	$\longrightarrow$	$(x \lor \neg z \lor \neg v \lor l_1)$
	$(z \lor y)$	$\longrightarrow$	$(z \lor y \lor l_2)$
	$(v \lor t)$	$\longrightarrow$	$(v \lor t \lor l_3)$
	(t)	$\longrightarrow$	$(t \lor l_4)$
Hard Clauses	$(y \lor t \lor \neg x)$	$\longrightarrow$	$(y \lor t \lor \neg x)$

Direct application of MaxSAT preprocessing

Conversion to LCNF adds labels to all soft clauses

MaxSAT Preprocessing can mix labels **LCNF**

Soft Clauses

$(x \lor y \lor t \lor l_1 \lor l_2 \lor l_3)$

$(t \lor l_4)$

Hard Clauses

$(y \lor t \lor \neg x)$

Direct application of MaxSAT preprocessing

Conversion to MaxSAT adds new variables and clauses

MaxSAT		LCNF
Soft Clauses	$(\neg l_1)$	$(x \lor y \lor t \lor l_1 \lor l_2 \lor l_3)$
	$(\neg l_2)$	
	$(\neg l_3)$	
	$(\neg l_4)$	$(t \lor l_4)$
Hard Clauses	$(x \lor y \lor t \lor l_1 \lor l_2 \lor l_3)$	
	$(t \lor l_4)$	
	$(y \lor t \lor \neg x)$	$(y \lor t \lor \neg x)$

Direct application of MaxSAT preprocessing

MaxSAT | | **LCNF**

Soft Clauses

$\cancel{(\neg l_1)}$ $(\neg l_1 \lor a_1)$

$\cancel{(\neg l_2)}$ $(\neg l_2 \lor a_2)$

$\cancel{(\neg l_3)}$ $(\neg l_3 \lor a_3)$

$\cancel{(\neg l_4)}$ $(\neg l_4 \lor a_4)$

MaxHS controls soft clauses by adding assumption variables

Hard Clauses

$(x \lor y \lor t \lor l_1 \lor l_2 \lor l_3)$

$(t \lor l_4)$

$(y \lor t \lor \neg x)$

Direct application of MaxSAT preprocessing

MaxSAT | **LCNF**

Soft Clauses

$\cancel{(\neg l_1)}$ $\cancel{(\neg l_1 \vee a_1)}$

$\cancel{(\neg l_2)}$ $\cancel{(\neg l_2 \vee a_2)}$

$\cancel{(\neg l_3)}$ $\cancel{(\neg l_3 \vee a_3)}$

$\cancel{(\neg l_4)}$ $\cancel{(\neg l_4 \vee a_4)}$

Hard Clauses

$(x \vee y \vee t \vee \mathbf{l_1} \vee \mathbf{l_2} \vee \mathbf{l_3})$

$(t \vee \mathbf{l_4})$

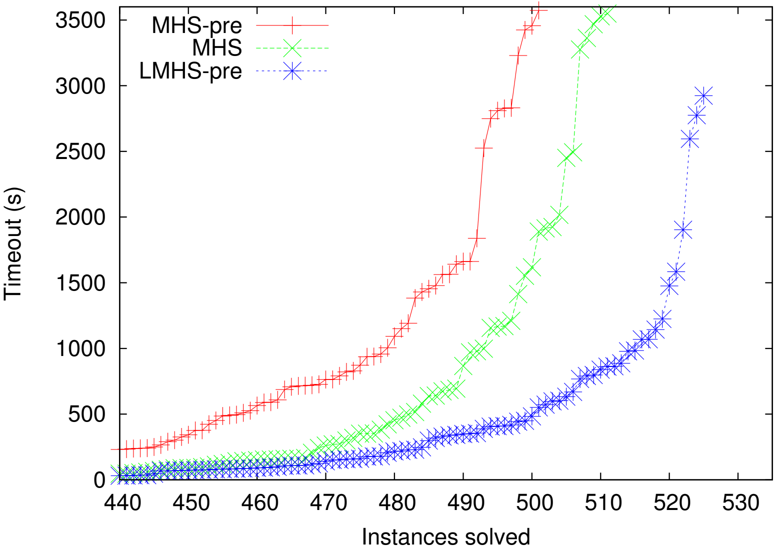
$(y \vee t \vee \neg x)$

Idea of Lifting
Reuse variables
introduced in
LCNF conversion
as assumptions

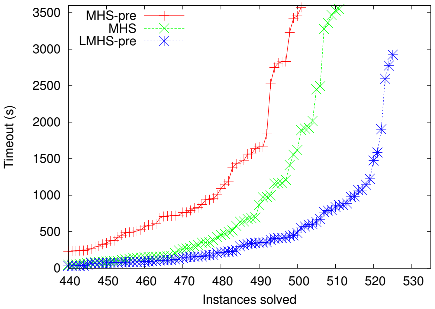
Lifting of MaxHS to LCNF:s

- Label centric reformulation of the MaxHS algorithm
  Davies and Bacchus [2013]
  - No extra assumption variables added during solving
    - Explicit list of assumption variables as input.
  - More than one assumption per clause allowed.

Experimental Evaluation



Conclusions



- Preprocessing MaxSAT can be done using LCNFs.
- Direct application degrades performance of the MaxHS algorithm.
- We propose a lifting of MaxHS to LCNF formulas.
- Tighter integration between MaxSAT preprocessing and solving.
- Improves performance over the original MaxHS algorithm.

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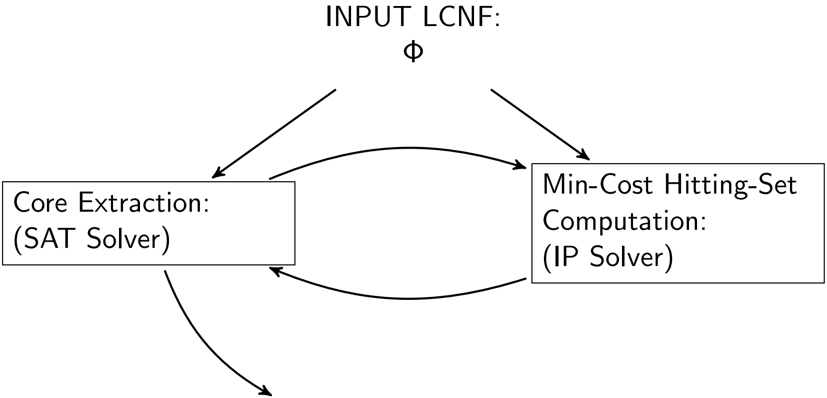
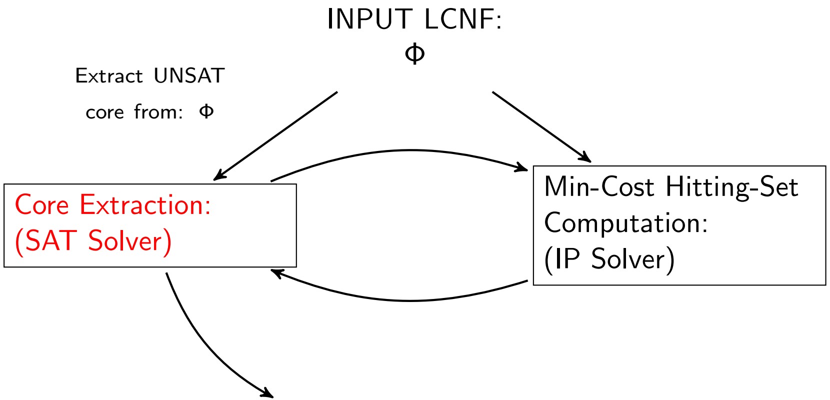
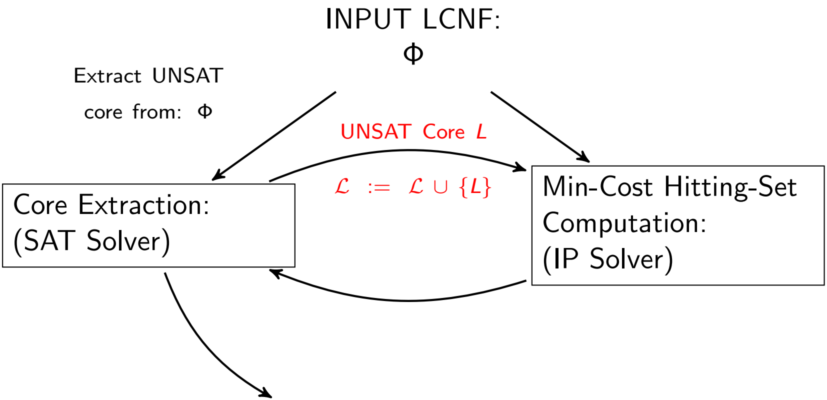
LCNF-MaxHS


INPUT LCNF:
Φ
Core Extraction:
(SAT Solver)
Min-Cost Hitting-Set
Computation:
(IP Solver)

LCNF-MaxHS


INPUT LCNF:
Φ
Extract UNSAT
core from: Φ
Core Extraction:
(SAT Solver)
Min-Cost Hitting-Set
Computation:
(IP Solver)

LCNF-MaxHS


INPUT LCNF:
Φ
Extract UNSAT
core from: Φ
UNSAT Core L
$\mathcal{L} := \mathcal{L} \cup \{L\}$
Core Extraction:
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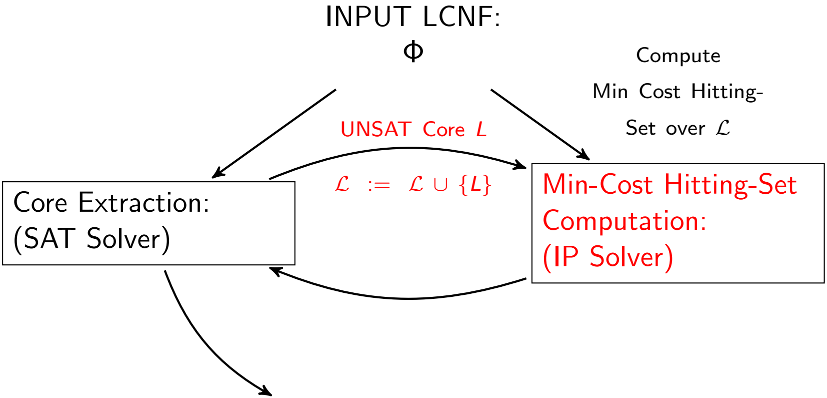
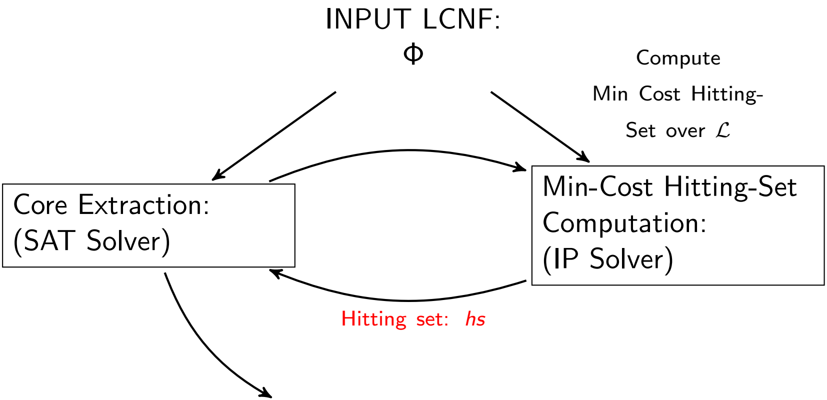
Min-Cost Hitting-Set
Computation:
(IP Solver)

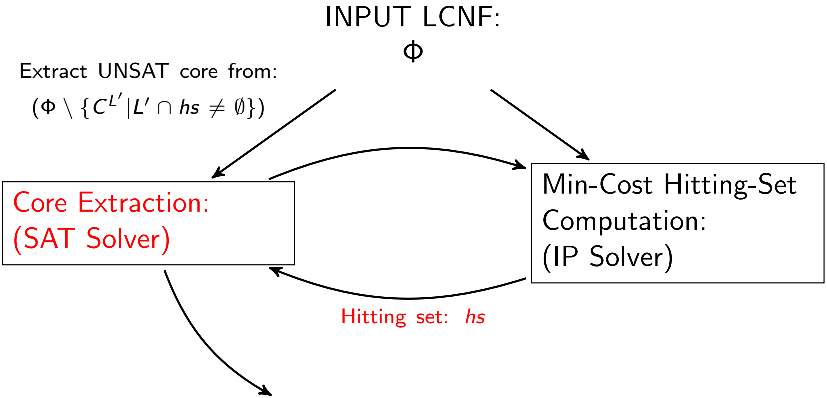
LCNF-MaxHS


INPUT LCNF:
Φ
Compute
Min Cost Hitting-
Set over $\mathcal{L}$
UNSAT Core L
$\mathcal{L} := \mathcal{L} \cup \{L\}$
Core Extraction:
(SAT Solver)
Min-Cost Hitting-Set
Computation:
(IP Solver)

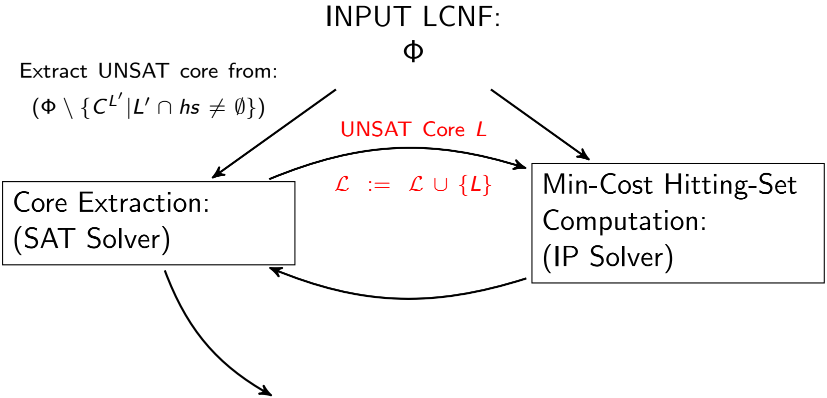
LCNF-MaxHS


INPUT LCNF:
Φ
Compute
Min Cost Hitting-
Set over $\mathcal{L}$
Core Extraction:
(SAT Solver)
Min-Cost Hitting-Set
Computation:
(IP Solver)
Hitting set: *hs*

LCNF-MaxHS


INPUT LCNF:
Φ
Extract UNSAT core from:
$(\Phi \setminus \{C^{L'} | L' \cap hs \neq \emptyset\})$
Core Extraction:
(SAT Solver)
Min-Cost Hitting-Set
Computation:
(IP Solver)
Hitting set: hs

LCNF-MaxHS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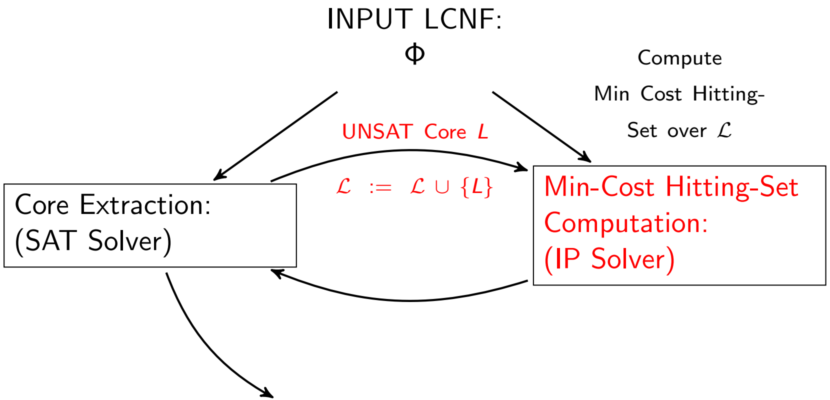
LCNF-MaxHS



INPUT LCNF:
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Compute
Min Cost Hitting-
Set over $\mathcal{L}$
UNSAT Core L
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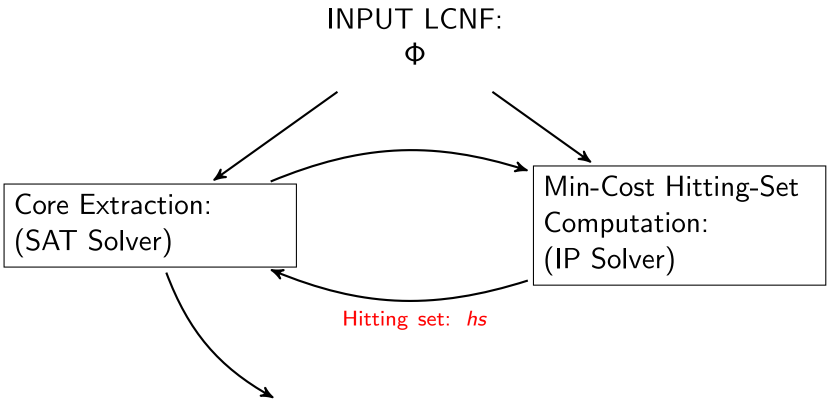
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Φ
Core Extraction:
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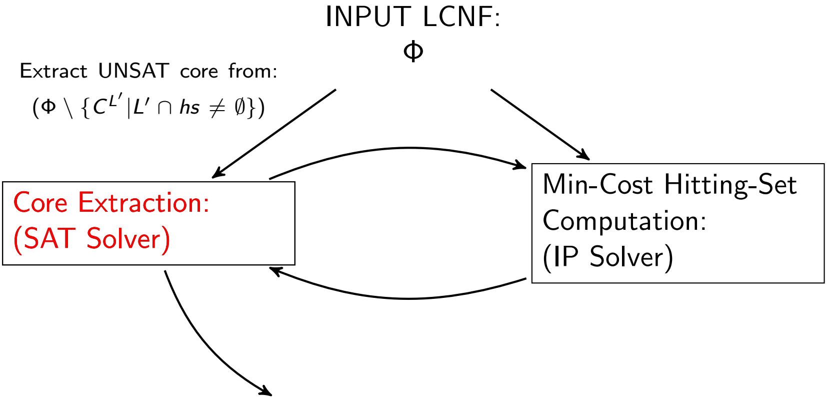
Min-Cost Hitting-Set
Computation:
(IP Solver)
Hitting set: *hs*

LCNF-MaxHS



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Φ
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Min-Cost Hitting-Set
Computation:
(IP Solver)

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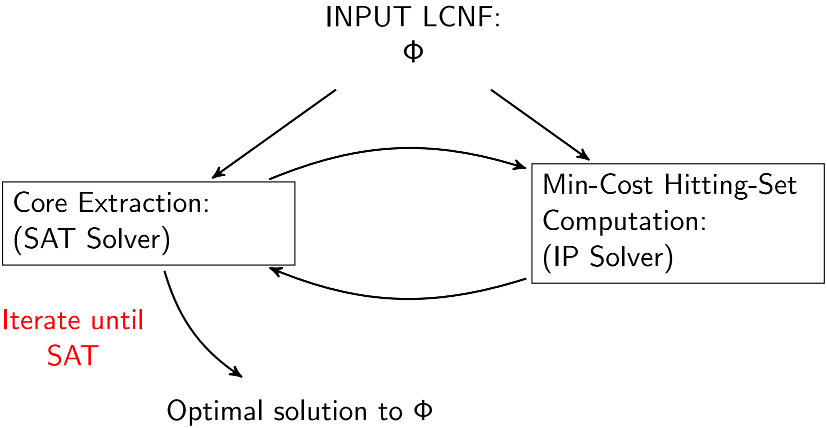
INPUT LCNF:
Φ
Core Extraction:
(SAT Solver)
Min-Cost Hitting-Set
Computation:
(IP Solver)
Iterate until
SAT
Optimal solution to Φ